

# A Course In Algebraic Number Theory

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## Preface

This is a text for a basic course in algebraic number theory, written in accordance with the following objectives.

1. Provide reasonable coverage for a one-semester course.
2. Assume as prerequisite a standard graduate course in algebra, but cover integral extensions and localization before beginning algebraic number theory. For general algebraic background, see my online text “Abstract Algebra: The Basic Graduate Year”, which can be downloaded from my web site [www.math.uiuc.edu/~r-ash/](http://www.math.uiuc.edu/~r-ash/) The abstract algebra material is referred to in this text as TBGY.
3. Cover the general theory of factorization of ideals in Dedekind domains, as well as the number field case.
4. Do some detailed calculations illustrating the use of Kummer’s theorem on lifting of prime ideals in extension fields.
5. Give enough details so that the reader can navigate through the intricate proofs of the Dirichlet unit theorem and the Minkowski bounds on element and ideal norms.
6. Cover the factorization of prime ideals in Galois extensions.
7. Cover local as well as global fields, including the Artin-Whaples approximation theorem and Hensel’s lemma.

Especially helpful to me in preparing this work were the beautiful little book by Samuel, “Algebraic Theory of Numbers”, Hermann 1971, and the treatment of cyclotomic fields by J. Milne in his online text “Algebraic Number Theory” ([www.math.lsa.umich.edu/~jmilne/](http://www.math.lsa.umich.edu/~jmilne/)) Some other useful references are:

Esmonde, J., and Murty, M.R., “Problems in Algebraic Number Theory”, Springer 1999  
 Frölich, A., and Taylor, M.J., “Algebraic Number Theory”, Cambridge 1991  
 Janusz, G.J., “Algebraic Number Fields”, AMS 1996  
 Marcus, D.A., “Number Fields”, Springer 1977  
 Stewart, I., and Tall, D., “Algebraic Number Theory”, Chapman and Hall 1987

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