Calculus Reorganization Proposal

Disclaimer

The following represents the views of the Undergraduate Affairs Committee in Mathematics and do not necessarily indicate a departmental endorsement or even general agreement unless indicated otherwise.

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History

In the Fall of 2004, the Executive Committee of the Mathematics Department gave the charge to the Undergraduate Affairs Committee (UAC) to examine the three advanced calculus courses offered, namely Math 242, 243 and 380, and to address redundancy and quality concerns that had been expressed by faculty. After several meetings, the Undergraduate Affairs Committee came to the position that one reasonable solution to these concerns was the elimination of Math 242 in favor of Math 243 and to completely eliminate Math 380 as it was.

One of the primary difficulties with this solution, however, was the fact that the addtional material in Math 243 which was not covered in Math 242 required additional meeting times and effort on the part of the students. The additional material requires that Math 243 is offered for 4 credits as opposed to the 3 credits for Math 242. As the majority of the students taking Math 242 are engineering students and nearly all the engineering curricula at UIUC have no room for additional credit units, this additional one credit represented a very serious problem. Many options were considered by the Undergraduate Affairs Committee and research was done as to how other programs handled these problems.

One possible solution to the additional credit problem that persisted was to find a way to offer some variant of the first semester calculus course, namely Math 220, for one less credit. Stephanie Alexander, chair of the UAC, and Randy McCarthy met with the Physics UAC to discuss these issues. It was felt that since Physics was one of the larger Engineering units to currently require Math 380, their input would be especially helpful at this preliminary stage. With their help the basic outlines of the final proposal was formed and they voted unanimously to support this change.

The UAC of mathematics again debated this new proposal and in particular examined its effect for good pedagogy, student enrollments, instructional obligations, and total effect on departmental instructions units. Randy McCarthy meet with the Engineering-Mathematics Liaison Committee on December 1. After several hours of discussion, many useful modificatons in terms of naming and numbering were incorporated into the basic proposal. As the Liaison Committee was enthusiastically in support of the overall proposal, the chair, Sri Namachchivaya, asked for a one page summary to send to the Engineering departments for further feedback.

Proposal for reorganizing Calculus

The Undergraduate Affairs Committee in Mathematics agreed on December 7, 2004 to distribute for comment a possible reorganization of the calculus sequences in Mathematics. The following is a one page summary of this proposal. You can find more information about this proposal at

http://www.math.uiuc.edu/~randy/CalcProp/

The Committee would like to know issues, concerns or praise you may have for this possible change.

Currently

Math 220 Calculus I (5 credits) 3 lectures/2 recitations

Math 230 Calculus II (3 credits) 2 lectures/2 recitations

Math 242 Calculus of Several Variables (3 credits) 2 lectures/2 recitations

Math 243 Calculus III Plus (4 credits) 4 lectures

Math 380 Advanced Calculus (3 credits) 3 lectures

Proposed

Math 220 Calculus I for Beginners (5 credits) 3 lectures/2 recitations
This is a renamed Math 220. Our current Math 220 assumes no previous exposure to calculus.

Math 221 Calculus I (4 credits) 2 lectures/2 recitations
This is Math 220 for those with previous exposure to calculus. That is, for students with a 2 or better on the AB advanced placement exam or a one year high school calculus course.

Math 231 Calculus II (3 credits) 2 lectures/2 recitations
This is a renumbered Math 230.

Math 236 Calculus with Applications (4 credits) 3 lectures/2 recitations
This is new course for science and engineering students with a 3 or better on the BC advanced placement exam. It will be co-created with engineering and team taught for two years after which the math department will run it independently. It is intended for students with a basic knowledge of calculus techniques but requiring the deeper understanding gained through applications.

Math 241 Calculus III (4 credits) 3 lectures/2 recitations
This is a renumbered Math 243 stressing 3 dimensions. In particular it will cover the vector integration theorems (Gauss, Green, Stokes and Divergence) currently taught in Math 243 and Math 380.

Scheduling The new Math 221 and Math 236 would be taught only in the fall. The audience for Math 221 is expected to be primarily engineering students. We currently teach approximately 300 engineering students Math 220 in the fall and we teach 15 in the spring.

Engineering: The recommended calculus sequence for engineering students would change as follows:

Currently: MA220, MA230, MA242 (11 credits total)
Proposed: MA221, MA231, MA241 (11 credits total)

However, the proposed sequence has an additional 16 hours on vector integration theorems compared to the sequences currently taught. Students in engineering who have campus credit for Math 221 and Math 231 (by way of a 3 or better on the BC advanced placement exam or junior college transfer) may be required by engineering to take Math 236 which is a review of the first year of calculus with a particular emphasis on problem solving techniques used in engineering.

Comparing the syllabi of 242, 243 and 380

The official departmental syllabi for these courses can be found at:

Math 242 (http://www.math.uiuc.edu/Bourbaki/Syllabi/syl242.html)
Math 243 (http://www.math.uiuc.edu/Bourbaki/Syllabi/syl243s.html)
Math 380 (http://www.math.uiuc.edu/Bourbaki/Syllabi/syl380.html)

Summary of Critical Points

Math 243 has 15 more scheduled meetings than Math 242.

Math 243		**Math 380**
-1	lectures 242 material	18
16	lectures on vector integration	16
0	lectures on Physical Appl.	5
0	added miscellaneous	5

Math 242. Calculus of Several Variables
Syllabus for Edwards and Penny Early Transendentals 6E

Chapter 11. Vectors, Curves, and Surfaces in Space (11 hours)

11.1 Vectors in the Plane (1)
11.2 Three Dimensional-Vectors (2)
11.3 The Cross Product of Two Vectors (1)
11.4 Lines and Planes in Space (1)
11.5 Curves and Motion in Space (2)
11.6 Curvature and Acceleration (2)
11.7 Cylinders and Quadratic Surfaces (1)
11.8 Cylindrical and Spherical Coordinates (1)

Hour Exam 1 (1 hour)

Chapter 12. Partial Differentiation (13 hours)

12.1 Introduction (.1)
12.2 Functions of Several Variables (.9)
12.3 Limits and Continuity (1)
12.4 Partial Derivatives (2)
12.5 Multivariable Optimization Problems (2)
12.6 Increments and Linear Approximation (1.5)
12.7 The Multivariable Chain Rule (1.5)
12.8 Directional Derivatives and the Gradient Vector (2)
12.9 Lagrange Multipliers and Constrained Optimization (1)
12.10 Critical Points of Functions of Two Variables (1)

Hour Exam 2 (1 hour)

Chapter 13. Multiple Integrals (13 hours)

13.1 Double Integrals (2)
13.2 Double Integrals over More General Regions (1)
13.3 Area and Volume by Double Integration (1)
13.4 Dougle Integrals in Polar Coordinates (1)
13.5 Applications of Double Integrals

(1)
13.6 Triple Integrals (2)
13.7 Integration in Cylindrical and Spherical Coordinates (2)
13.8 Surface Area (1)
13.9 Change of Variables in Multiple Integrals (2)

Hour Exam 3 (1 hour)

Leeway (3 hours)

Total Hours (43 hours)

Math 243. Calculus III Plus
Syllabus for Lecture-Discussion Form of Math 243

Text: Marsden, Tromba, and Weinstein, *Basic Multivariable Calculus*, W. H. Freeman & Company, 1993.

This four hour course is an alternative to the three hour course Math 242, Calculus of Several Variables. It enhances the standard course by including the integral theorems of vector analysis and applications to electromagnetics and fluid mechanics.

- Chapter 1. Algebra and Geometry of Euclidean Space (10 days)
  - 1.1 Vectors in the Plane and Space
  - 1.2 The Inner Product and Distance
  - 1.3 2x2 and 3x3 Matrices and Determinants
  - 1.4 The Cross Product and Planes
  - 1.5 n-Dimensional Euclidean Space
  - 1.6 Curves in the Plane and in Space
- Chapter 2. Differentiation (10 days)
  - 2.1 Graphs and Level Surfaces
  - 2.2 Partial Derivatives and Continuity
  - 2.3 Differentiability, the Derivative Matrix, and Tangent Planes
  - 2.4 The Chain Rule
  - 2.5 Gradients and Directional Derivatives
  - 2.6 Implicit Differentiation
- Chapter 3. Higher Derivatives and Extrema (5 days)
  - 3.1 Higher Order Partial Derivatives
  - 3.2 Taylor's Theorem
  - 3.3 Maxima and Minima
  - 3.4 Second Derivative Test
  - 3.5 Constrained Extrema and Lagrange Multipliers
- Chapter 4. Vector-Valued Functions (4 days)
  - 4.1 Acceleration
  - 4.2 Arc Length
  - 4.3 Vector Fields
  - 4.4 Divergence and Curl
- Chapter 5. Multiple Integrals (10 days)
  - 5.1 Volume and Cavalieri's Principle
  - 5.2 The Double Integral Over a Rectangle
  - 5.3 The Double Integral Over Regions
  - 5.4 Triple Integrals
  - 5.5 Change of Variables, Cylindrical and Spherical Coordinates
  - 5.6 Applications of Multiple Integrals
- Chapter 6. Integrals over Curves and Surfaces (6 days)
  - 6.1 Line Integrals
  - 6.2 Parmetrized Surfaces
  - 6.3 Area of a Surface
  - 6.4 Surface Integrals
- Chapter 7. The Integral Theorems of Vector Analysis (7 days)
  - 7.1 Green's Theorem
  - 7.2 Stokes's Theorem
  - 7.3 Gauss's Theorem
  - 7.4 Path Independence and the Fundamental Theorems of Calculus
- Exams (4 days)
- Leeway (2 days)
- Total (58 days)

Math 380: Advanced Calculus (formerly Math 280)
Syllabus for Instructors

Text: W. Kaplan, *Advanced Calculus,* 5th Edition, Addison Wesley Longman, 2003.

Chapter		Class Hours
2	Differential Calculus of Functions of Several Variables	
	Sections 1-6	2
	Sections 7-8	2
	Sections 9-12	3
	Sections 13-18	3
	Sections 19-20	2
3	Vector Differential Calculus	
	Sections 1-6	2
4	Integral Calculus of Functions of Several Variables	
	Sections 3-7, 9	5
5	Vector Integral Calculus	
	Sections 1-13	14
	A selection of physical applications	
	Sections 14-18 (these may be integrated with the above topics)	5
	Leeway and Exams	5
	Total	43

NOTE: This course is aimed at students primarily interested in the applications of mathematics. To the extent possible, proofs should be replaced by intuitive explanations and plausible manipulations. Applications of the material should be given, at the very least, to heat conduction and Maxwell's equations. For example, one should explain how Maxwell equations lead to the wave equation for electromagnetic waves, and hence predict electromagnetic radiation.

Third Semester Calculus at some other institutions

Prepared by Randy McCarthy, Fall 2004

The mathematics department at UIUC currently offers a basic three semester calculus sequence. The third course is multivariable calculus. Presently, most students take Math 242 as their third course. However, this 3 credit course does not reach the "integral theorems" (in particular Green's Theorem, Stoke's Theorem or the Divergence Theorem). The national standard for third semester calculus is quickly becoming a course which includes these results. In fact, the California Articulation agreement has made this the standard since 1995 for even the junior colleges.

CAN: MATH 22

Title: Calculus, 3rd semester

Description: Vector-valued functions; calculus of functions of more than one variable, partial derivatives, multiple integration; Green's theorem, and the divergence theorem.

The department of mathematics at UIUC has run such a third semester course, Math 243, since the spring of 1998. However, because of the additional material it needs to be offered for 4 credits. The department offers a course, Math 380, which is a fourth semester calculus course for 3 credits which also contains the integral theorems. However, this course has a great deal of overlap with Math 242 and is seen as unnecessarily redundant.

Proposal: The elimination of Math 242 and Math 380. Math 243 will be taught in large lectures 3 days a week with recitations 2 days a week for 4 credits.

The department of mathematics is sympathetic that the additional material may make third semester more difficult for some students. In particular, the additional credit hour is going to cause difficulties for programs (especially in engineering) which already have very few available credits for electives. However, UIUC currently receives one of the lowest number of credits for the three semester calculus sequence. In particular, third semester calculus is essentially always given 4 to 5 semester credit units and the primary difference in some programs (like Michigan State University or MIT) occurs in the first semester calculus course.

Stanford, 13.3 credits, [41, 42, 51, 52 (5, 5, 5, 5) quarter system 13.3 credits]

Berkeley, 12 credits, [1A, 1B, 53 (4, 4, 4)]

University of Michigan, 12 credits, [110, 115, 215 (4, 4, 4)]

Michigan State University, 11 credits, [132, 133, 234 (3, 4, 4)]

Some engineering departments offer the calculus in an accelerated format for fewer credits. For example.

Cal Tech, 9 credits

> They receive 27 units over three quarter courses. Full time at Cal Tech is 36 hours so one might divide by 3 to get UIUC equivalent which would be only 9 credits. However, it is taught over one year, not a year and a half. This is because the course begins with a review of calculus and does not assume a student is new to the topic.

Northwestern University, Engineering Sciences and Applied Math, 8 credits

> They also offer a three sequence Math 214-1,2,3 for 4,4,4 credits. These are quarter credits so equivalent to 8 semester credits. They, like MIT below, however, do not cover sequences and series and other topics that we do in Math 230.

MIT, 8 credits

> They offer several variations but we will stick to their primary sequence. Math 18.01 is offered for the equivalent of 4 semester credits and is their first calculus course. Their second semester course is Math 18.02 and also offered for the equivalent of 4 semester credit hours.
>
> > **MATH 18.02**
> > Prereq: 18.01
> > Units: 5-0-7 (12 total, 3 units = 1 semester credit)
> > **Description:** Double integrals and line integrals in the plane; exact differentials and conservative fields; Green's theorem and applications, triple integrals, line and surface integrals in space, Divergence theorem, Stokes' theorem; applications.

A comparison of their Math 18.01 with our Math 220, 230 courses is as follows:

topic	# lec. MIT	# lec. UIUC
derivatives and applications	14	23
integrations and applications	7	14
techniques of integration	4	6
polar coordinates and integration	3	7
infinite series, Taylor's	2	10
Total Lectures	30	60

Why create Math 221?

Math 220 at UIUC assumes NO previous experience with calculus. A greater number of incoming students to UIUC have, however, already seen some calculus. It is not uncommon for students who already have Advanced Placement (AP) credit for the first semester course Math 220 to take it anyhow. The reasons vary, sometimes the students are hoping for 5 credits of an easy good grade, but often the students are self-aware that their grasp of the material is weak despite managing to do well on a placement examination. These students are not always a positive influence for the students who have seen little to none of the material presented in math 220 before.

Another group of students have seen a large portion of Math 220 in high school but were either not offered an AP calculus course or missed their chance to take the exam. One major problem for these students is boredom. There is a high degree of repetition in the 3 hours of lecture for Math 220 as many of the concepts are very difficult for first time students. Those who have seen much of the material before become bored early in the course and spend little time on it. Later, when the material becomes more difficult or new for these students, they are unprepared to do the additional work the course demands and actually do worse than one would have expected.

Math 221 will cover the SAME syllabus at that of Math 220, but with 2 lectures per week instead of 3. There will still be 2 recitations a week for doing the exercises. The expected audience for Math 221 are engineering students with some calculus experience (as their program will require it) and students with some confidence in their mathematical skills but wanting to brush up on their calculus at a college level. Having no lecture to attend on Friday will appeal to some students and a faster paced coverage of the material on Mondays and Wednesdays may help with boredom. As this audience exists primarily in the fall, math 221 will not be offered in the spring or summer.

What is Math 236?

Math 236 is a concept (most likely not new) by Douglas Beck from Physics and Randy McCarthy to address a common complaint by faculty in Engineering. They are finding that a growing percentage of their students are obtaining advanced placement credit equivalent to both Math 220 and Math 230 (either by taking the BC exam or by transer credit) but their math understanding remains weak. That is, these students are technically able to solve straightforward calculus problems but their ability to do word problems or other applications of these same techniques is poor. It was felt that a one semester review of the first year calculus course stressing engineering applications would serve these students better than making them repeat Math 230 (which is currently their only option) or rushing into math 242 unprepared.

We would like the new Math 236 to be created jointly by both math faculty and those from engineering. It should also be team taught with faculty from both departments for the first two years (at least). In this way, we can be sure to have a good balance between

the mathematical understanding (primarily through the lectures) and the engineering applications (primarily through the recitations). This course should be taught at a level quite a bit higher than a usual first year calculus course and the students who finish with math 236 would then take math 241. Math 236 would be offered only in the fall and it is expected to serve between 100 and 150 students the first year (one large lecture). However, if successful, given the national trend for more highschools attempting to teach AP calculus, math 236 would be expected to grow in enrollment over time.

Why renumber and rename some courses?

When Randy McCarthy met with the Engineering-Mathematics Liaison Committee, it was stressed that the naming and numbering of the newly reorganized calculus sequence would be important for students, advisors and faculty.

Since there would be two variants of math 220 in the new system, a lot of discussion focused upon how to properly number and name these. In addition, some members of the UAC felt that the standard math sequence should appear more sequential (for example, to have 220, 230, 240 instead of 220, 230 and 242 or 243). Since math 220 is required or recommended by many divisions on campus it was felt that changing its number would cause unnecessary confusion.

A solution which seemed to resolve these problems was to simply renumber the new sequence 221, 231 and 241. Of course, this would become the recommended sequence in Engineering, while 220/221 and 231 would most likely be the recommended one year calculus sequence for non-engineers. The only artificial change is the renumbering of math 230 to math 231. Some short term trouble will occur by this change, but it was felt that it would not be serious.

Some debate focussed upon the numbering of the primary sequence. There were two logical choices, either (220, 230, 240) or (221, 231, 241). There was a desire to have the sequence reflect that freshmen at UIUC often have some previous exposure to calculus. In addition, it was felt that if one used the sequence (220, 230, 240), Math 221 would appear as an accelerated or advanced course. If one uses the sequence (221, 231, 241) then Math 220 appears as a slightly less intense start to the standard sequence, which is indeed what Math 220 is supposed to be.

The new course, Calculus with Applications, was suggested to be numbered 236 for a couple reasons. It was felt that its number should be greater than 231 and less than 241. Math 240 was suggested, but since 240 is not a gentler version of Math 241 (as Math 220 is to Math 221) it was felt some confusion might arise if 240 were used. In recent history, Math 235 existed as a different accelerated calculus course and so one did not want the new course confused with this. Thus, Math 236 was chosen, as simply the average of 231 and 241 and having no obvious historical counterpart to cause confusion.

Naming the new courses was cause for much spirited discussions both in the Engineering Liason Committee and by members of the Mathematics UAC. It was generally agreed

that calling the new Math 221 some sort of "accelerated" name would discourage enrollment. At the same time, one wanted it made clear that the content of both Math 220 and 221 was the same, and only the expect level of the enrolled students differed. Thus, the title “Calculus I for Beginners” for Math 220 and “Calculus I” for the new Math 221 was suggested.