

# Math 220–Test 2

University of Illinois, October 17, 2008

**NAME:** \_\_\_\_\_

**SECTION:** \_\_\_\_\_

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**No calculators, notes, text or phones during the exam.**

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**1.** (6) \_\_\_\_\_

**2.** (24) \_\_\_\_\_

**3.** (6) \_\_\_\_\_

**4.** (6) \_\_\_\_\_

**5.** (6) \_\_\_\_\_

**6.** (6) \_\_\_\_\_

**7.** (6) \_\_\_\_\_

**8.** (10) \_\_\_\_\_

**Total.** (70) \_\_\_\_\_

1. Limits. *6 points, 2 points each*

Compute the following limits. Indicate your reasoning.

1a.

$$\lim_{x \rightarrow \infty} \frac{3x^4 + 2x - 4}{78x^3 + 50x^2 + x}$$

1b.

$$\lim_{x \rightarrow \infty} \frac{3x^3 + 2x - 4}{78x^3 + 50x^2 + x}$$

1c.

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 2x - 4}{78x^3 + 50x^2 + x}$$

2. Derivatives. *24 points, 4 points each*

Find the derivative of each function.

2a. (HW)

$$f(t) = \sin(t) \sec(t)$$

2b. (HW)

$$f(x) = \ln(\cos(x))$$

Problem 2 continued.

**2c.** (HW)

$$f(x) = \frac{e^{4x}}{x}$$

**2d.**

$$f(x) = \sin^{-1}(x^3 + 1)$$

**2e.**

$$f(x) = \sqrt{2 + \tan^{-1}(x)}$$

**2f.**

$$f(t) = (2 - t)^{120}$$

**3.** (HW) Logarithmic derivative. *6 points*

Use logarithmic differentiation to find the derivative.

$$f(x) = x^{\sin x}$$

**4.** Implicit differentiation. *6 points*

Find an equation of the tangent line at the given point.

$$x \sin(y) + y \cos(x) = \frac{\pi}{2} \text{ at } (0, \frac{\pi}{2})$$

**5.** Inverse functions. *6 points*

The function  $f(x)$  has an inverse  $g(x)$ . Find  $g'(a)$  if

$$f(x) = x^5 + 4x - 2, a = -2.$$

**6.** Theorems. *6 points*

Using theorems from class, show that the function  $f(x) = x^3 - 6x + 2$  has a root between  $-1$  and  $1$  and furthermore that it has exactly one such root between  $-1$  and  $1$ .

**7.** Interpretation of derivatives. (HW) *6 points*

A spring hanging from the ceiling vibrates up and down. Its vertical position at time  $t$  is given by  $f(t) = 4 \sin 3t$ . Find the velocity of the spring at time  $t$ . What is the spring's maximum speed? What is its location when it reaches its maximum speed?

**8.** Related rates. *10 points* (HW)

A dock is 6 feet above water. Suppose you stand on the edge of the dock and pull a rope attached to a boat at the constant rate of 2 ft/s. Assume that the boat remains at water level. At what speed is the boat approaching the dock when it is 20 feet from the dock?