Math 220–Test 3

University of Illinois, November 14, 2008

NAME: ____________________

SECTION: ____________________

No calculators, notes, text or phones during the exam.

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1. (4) ___________

2. (8) ___________

3. (8) ___________

4. (10) ___________

5. (10)___________

6. (10) ___________

7. (10) ___________

8. (10) ___________

Total. (70) ___________

1. (HW) Newton's Method *(4 points)*

Use Newton's method (one iteration will suffice) to estimate $\sqrt{11}$ starting with an initial guess of $x_0 = 3$.

2. (HW) L'Hôpital's Rule *(8 points, 3/5)*

2a.

$$\lim_{x \to 0} \frac{\tan^{-1} x}{\sin x}$$

2b.

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

3. Integration *(8 points, 3/5)*

3a. (HW) Find the derivative $f'(x)$

$$f(x) = \int_0^{x^2} \left(e^{-t^2} + 1\right) \, dt$$

3b. Show that the two shaded regions have the same area:

4. Graph sketching *(10 points, 2/1/2/2/3)*

Consider the function:

$$f(x) = \frac{1}{1+x^2} \qquad f'(x) = \frac{-2x}{(1+x^2)^2} \qquad f''(x) = \frac{6x^2-2}{(1+x^2)^3}$$

4a. Discuss any horizontal or vertical asymptotes that f possesses.

4b. When does the graph of f cross the x-axis and the y-axis?

4c. Where is the function increasing and decreasing? If the function has any local max or minimums, what are they?

4d. Where is the function concave up and concave down? If f has any inflection points, what are they?

4e. Sketch a graph of the function on the axes provided. If f has a global maximum or minimum, indicate them on the graph.

5. Optimization *10 points*

Find the point on the curve $y = \sqrt{x}$ closest to the point $(1, 0)$.

6. Graph sketching *10 points, 1/3/3/3*

Consider the function:

$$\begin{aligned} f(x) &= x^4 - 4x^3 + 5x^2 - 2x \quad &&= x(x-1)^2(x-2) \\ f'(x) &= 4x^3 - 12x^2 + 10x - 2 &&= (x-1)(x-(1+\frac{\sqrt{2}}{2}))(x-(1-\frac{\sqrt{2}}{2})) \\ f''(x) &= 12x^2 - 24x + 10 &&= (x-(1+\frac{\sqrt{6}}{6}))(x-(1-\frac{\sqrt{6}}{6})) \end{aligned}$$

where we recall that $0 < \frac{\sqrt{6}}{6} < \frac{\sqrt{2}}{2} < 1$.

6a. When does the graph of f cross the x-axis?

6b. Where is the function increasing and decreasing? If the function has any local max or minimums, what are they?

6c. Where is the function concave up and concave down? If f has any inflection points, what are they?

6d. Sketch a graph of the function on the axes provided. It may be helpful to know that $f(1-\frac{\sqrt{2}}{2}) = \frac{-1}{4}$ and $f(1+\frac{\sqrt{2}}{2}) = \frac{-1}{4}$.

7. (HW) Optimization *10 points*

Suppose a painting hangs on a wall as in the figure. The frame extends from 6 feet to 8 feet above the floor. A person whose eyes are 5 feet above the ground stands x feet from the wall and views the painting, with a viewing angle A formed by the ray from the person's eye to the top of the frame and the ray from the person's eye to the bottom of the frame. Find the value of x that maximizes the viewing angle A. [Hint: $A = \beta - \alpha$ in the picture]

8. Riemann Sums *10 points, 3/2/5* Consider the following integral

$$\int_1^7 -2x + 10\, dx$$

8a. Sketch the region of the plane represented by the integral on its graph and calculate the value of the integral using geometry.

8b. Use the Fundamental Theorem of Calculus to evaluate the integral.

8c. Use the *definition* of the integral via Riemann Sums (using the right end-point approximations) to evaluate the integral.