A DIAGRAM SKETCH OF THE FUNDAMENTAL THEOREM OF CALCULUS

The Fundamental Theorem of Calculus states that if f f is continuous on $[a, b]$, then

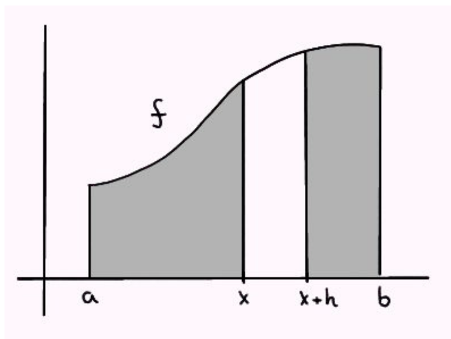
$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x).$$

Definition of Derivative.

By the definition of the derivative,

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = \lim_{h\to 0}\frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h} = \lim_{h\to 0}\frac{\int_x^{x+h} f(t)dt}{h}$$

since

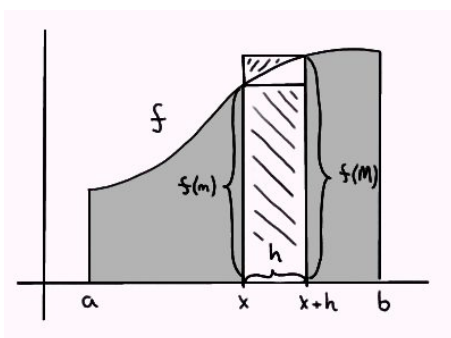


Extreme Value Theorem.

By the Extreme Value Theorem, the function f has a maximum value M and a minimum value m in $[x, x+h]$ for each $h > 0$. So

$$h \cdot f(m) \leq \int_x^{x+h} f(t)dt \leq h \cdot f(M)$$

since



Algebra.

Since $h \neq 0$, we can divide by h giving

$$f(m) \leq \frac{1}{h}\int_x^{x+h} f(t)dt \leq f(M).$$

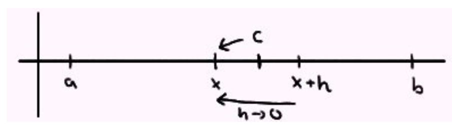
Intermediate Value Theorem.

By the Intermediate Value Theorem, we have that there is a point c in the interval $[x, x+h]$ such that

$$f(c) = \frac{1}{h}\int_x^{x+h} f(t)dt.$$

Limit of average values c.

As h goes to 0 the c must go to x



and because f is continuous,

$$\begin{aligned} f(x) &= \lim_{h\to 0} f(c) \\ &= \lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)dt \\ &= \lim_{h\to 0} \frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h} \\ &= \frac{d}{dx}\left(\int_a^x f(t)dt\right) \end{aligned}$$