

**Math 406, Exam I**  
University of Illinois, September 2006

**Instructions:** Please answer the questions as clearly as possible and ask if you are unsure about what is needed for full credit in your solution. No calculators, cheat sheets, pets, friends, family, enemies or psychic readings can be used to aid you with this exam. Good luck and remember that not all questions are weighted equally.

**1.** (*12 points*) The following are short answers and worth 2 points each.

**1a.** \_\_\_\_\_ is usually credited to have first “proved” that the ratio of the areas of two circles is proportional to the ratio of their squares. He used the same idea to calculate the area of a particular “lune.” Aristotle later claimed that this lune calculation was false.

**1b.** \_\_\_\_\_ used his method of exhaustion to give the first rigorous proof that the ratio of the areas of two circles is proportional to the ratio of their squares.

**1c.** Archimedes used both exhaustion and \_\_\_\_\_ to give the first complete proof that for a circle, the ratio of the area to the square of the radius was equal to the ratio of the circumference to the diameter of the circle.

**1d.** Archimedes made the \_\_\_\_\_ axioms which indicated in particular that under favorable conditions the problem of deciding which of two paths starting and ending at the same point in the plane is longer could be done by comparing areas they bounded.

**1e.** In his work *The Method*, Archimedes indicates a method to find the area of regions by using the “law of the \_\_\_\_\_.”

**1f.** The Pythagoreans believed that any two lengths were \_\_\_\_\_, which meant they are both integral multiples of a common length.

**2.** (*8 points*) Essay question

Al-Khwarizmi was an Arab astronomer and text book writer who combined three types of mathematical cultures in his work. What were these three cultures and what did they contribute to mathematics as we study it today?

**3.** (*8 points*) Prove the Pythagorean theorem using paper-cutting techniques like those of the Babylonians.

4. (10 points) Apply (Eudoxus') definition of proportionality to show that

$$a : b = c : d \quad \text{implies} \quad na : nb = mc : md$$

for any two (positive) integers  $m$  and  $n$ .

5. (12 points) Do ONE of the following two problems. If you work on both, indicate CLEARLY which attempt you wish to have graded otherwise you will receive the lower of the two attempts.

**5-Option 1.** Apply a slicing construction like that used (by Archimedes) in *On Conoids and Spheroids* to prove the formula for the volume of a (square based) pyramid.

**5-Option 2.** Let  $A$  be the area of a circle of radius  $r$  and let  $C$  be its circumference. Show that  $A$  cannot be bigger than  $\frac{1}{2}rC$  by approximating the circle with an inscribed regular polygon.