

Math 406, Exam I Review
University of Illinois, October 2004

Instructions: Please answer the questions as clearly as possible and ask if you are unsure about what is needed for full credit in your solution. No calculators, cheat sheets, pets, friends, family, enemies or psychic readings can be used to aid you with this exam. Good luck and remember that not all questions are weighted equally.

1. (12 points) The following are short answers and worth 2 points each. Example of a few I didn't use are:

Given a square, the length of one side and its diagonal are not _____ which brought into doubt some of the results of the _____ as they assumed this as part of their mathematics.

_____ proving mathematical results.

He was the first to claim that the ratio of areas of two similar regular polygons is equal to the ratio of the squares of one of their sides.

_____ implicitly used the Archimedian principle to justify the method of exhaustion.

His solution to the issue of incommensurable magnitudes amounts today to the fact that every real number can be approximated arbitrarily close by a rational number.

Al-Khwarizmi's textbook *Al-jabr wa'l muqabalah* has provided us with the modern word

Eureka! _____

_____ was first to calculate the quadrature of the parabola.

His notion of proportionality allowed one to work with ratios of arbitrary lengths. _____

Al-Khwarizmi was an Arab text book writer and astronomer who combined three types of mathematical notation in his work. These three elements were _____.

2. Essay question, 8 points

Explain how Eudoxus' definition of proportionality allowed the Greek's to work with ratios of arbitrary lengths even though they only allowed positive integers as "numbers".

3. (10 points) (I, 16)

Consider two rectangles with bases a and b , area A and B , that have the same height. Apply Eudoxus' definitions of proportionality to File Edit Options Buffers Tools TeX Help show that $A : B = a : b$.

4. (10 points) (II, 3)

Let A_n and C_n denote the area and perimeter, respectively, of a regular polygon with n sides inscribed in a circle of radius r . Show that

$$A_n = nr^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} \quad \text{and} \quad C_n = 2nr \sin \frac{\pi}{n}.$$

Deduce that $A = \frac{1}{2}rC$ by taking the limit of A_n/C_n as $n \rightarrow \infty$.

5. (10 points) (II, 23)

Let P be the segment of a paraboloid obtained by revolving the parabola $y^2 = x$, $0 \leq x \leq 1$, about the x -axis. Use Archimedes' mechanical method to deduce that the volume of P is one-half that of the circumscribed cylinder Z obtained by revolving the line segment $y = 1$, $0 \leq x \leq 1$, about the x -axis.