

Math 406, Exam II
University of Illinois, October 2006

Instructions: Please answer the questions as clearly as possible and ask if you are unsure about what is needed for full credit in your solution. No calculators, books, cheat sheets, pets, acquaintances, friends, family, enemies, magic, psychic readings or chanting can be used to aid you with this exam. Good luck and remember that not all questions are weighted equally.

1. (10 points) The following are short answers and worth 2 points each.

1a) In trying to explain the motion of the planets, his confusion about exhaustion techniques led him to claim that in polar coordinates, $A = \frac{1}{2} \int r \, ds$ when in fact $A = \frac{1}{2} \int r^2 \, d\theta$.
_____.

1b) He was the first to conjecture $\int_0^a x^n = \frac{a^{n+1}}{n+1}$ for n a positive integer which was later proved by Fermat, Descartes and others.

1c) He used a recursive method for determining the coefficients of $(x + y)^n$ for n a positive integer which resulted in a triangular array of numbers that now bears his name.

1d) _____ Theorem says that if a differentiable function f has a max or min at c then $f'(c) = 0$ though he never gave a clear reason for how he obtained his derivatives for polynomials.

1e) If $L(xy) = L(x) + L(y)$ for all $x, y > 0$ and $L(8) = 3$, what function is $L(x)$?

2. (12 points) Essay Question.

Describe the historical development of logarithms. Be sure to indicate the roles played by Napier, Briggs, Gregory, Newton and Mercator.

3. (8 points) (V, 5)

Apply the Cartesian circle method using Hudde's rule to show that the slope of the tangent line to $y = (x^2 + 1)^{3/2}$ is $3x(x^2 + 1)^{1/2}$.

4. (10 points) (IV, 7)

A spherical ring is obtained from a solid sphere by boring out a cylindrical hole whose axis is the vertical diameter of the sphere. Find the volume of the spherical ring by comparing it with a sphere whose diameter is equal to the height of the ring.

5. Do ONE of the following (10 points)

5a) Using modern first year calculus results, prove Newton's binomial coefficient theorem (convergence for $|x| < 1$).

5b) Using the limit $\lim_{n \rightarrow \infty} \frac{1}{n^{k+1}} \sum_{i=1}^n i^k$, prove that $\int_0^a x^n dx = \frac{a^{n+1}}{n+1}$.