Math 406, Practice for Exam II
University of Illinois, October 2006

Instructions: Please answer the questions as clearly as possible and ask if you are unsure about what is needed for full credit in your solution. No calculators, books, cheat sheets, pets, acquaintances, friends, family, enemies, magic, psychic readings or chanting can be used to aid you with this exam. Good luck and remember that not all questions are weighted equally.

1. (*20 points*) The following are short answers and worth 2 points each. (There will be five of these on the exam for 10 points total)

_____ used Napier's log tables to calculate planet positions.

_____ discovered the laws of planetary motion.

His technique for finding volumes of solids was by slicing the into tiny pieces he called "solid indivisibles" of a convenient shape and to add these up in an unspecified ad hoc way. _____

In the first of his two influential books he established a way to calculate the volumes of solids by comparing an unknown region with a known one by parallel slices having the same area. _____

_____ was the first to solve max-min problems by taking into account the characteristic behavior of a function near its extreme values though he never clearly explained his methods.

Descartes' _____ gave an algebraic method for constructing tangent lines.

_____ rule is the method suggested by Hudde's rule for finding the slope of the tangent line to an implicitly defined function.

Between 1594 and 1614 he created a computational method whose impact on science was argumentatively as important as that of the computers today. _____

The objective of Napier's logarithms was to reduce _____ to _____.

He calculated $\mathrm{Log}(n)$ for positive integers $1 \leq n \leq 20,000$ to 14 decimal accuracy. _____

Gregory St. Vincent's observation that $A_{a,b} = A_{ta,tb}$ for $t > 0$ was for what $A_{a,b}$? _____

If $L(xy) = L(x) + L(y)$ for all $x, y > 0$ and $L(9) = -1$, what function is $L(x)$?

Mercator's series for $\log(1+x)$ is _____

$\binom{\frac{1}{2}}{3}$ is _____

State Newton's binomial series: _____

His rule provides a convenient means of finding double roots to polynomials.

_____

He was the first to define the *natural* logarithm and proved a power series expansion for $\ln(1+x)$ which was earlier suggested by work of Newton.

_____

$\lim_{n\to\infty} \frac{1}{n^{k+1}} \sum_{i=1}^{n} i^k =$ _____.(Hint. Be sure your answer works for n=1, 2, 3)

He published tables in 1614 and instructions on how to use them to do calculations to high accuracy easily. They were immediately used by Kepler to establish his third rule of planetary motion.

_____

2. Essay Question (*12 points*)

Describe the historical development of the algebraic formulations of the derivative. Be sure to indicate the roles played by Fermat, Descartes, Hudde, Sluse and Barrow.

3. (*8 points*)

a.

Apply the Cartesian circle method using Hudde's rule to show that the slope of the tangent line to $y = (x^2 + 2)^{3/2}$ is $3x(x^2 + 2)^{1/2}$.

b.

Use Sluse's rule to show that the slope of the tangent line to $y = (x^2+2)^{3/2}$ is $3x(x^2+2)^{1/2}$.

4. (*10 points*)

Use Cavalieri's Principle to show that the regions bounded by the two curves have the same area.

$$y = 2x - x^2, y = 0 \qquad \text{and} \qquad y = x^2, y = 2x$$

4 (alt). (*10 points*)

Use Cavalieri's Principle to show that the volume obtained by rotating the region

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \quad y \geq 0$$

about the y–axis is equal to the volume of cylinder of radius a and height b minus an inverted circular cone of radius a and height b.

5. (*10 points*) (This will actually be a choose one of two problems related to the material)

5a) Prove that $p(x) = \binom{x}{n}$ is the unique polynomial of degree n such that $p(k) = 0$ for integers $0 \leq k \leq n - 1$ and $p(n) = 1$.

5b) Prove by Archimedes' method of compression that $A_{a,b} = A_{ta,tb}$ if $A_{a,b} = \int_a^b \frac{1}{x} dx$.