Math 406 Exam III
University of Illinois, December 2006

Instructions: Please answer the questions as clearly as possible and ask if you are unsure about what is needed for full credit in your solution. No computers, cell phones, calculators, books, cheat sheets, pets, acquaintances, friends, family, enemies, magic, psychic readings or chanting can be used to aid you with this exam. Good luck and remember that not all questions are weighted equally.

1. (*14 points*) The following are short answers and worth 2 points each.

Erhard Weigel was a professor of math and philosophy. During Leibniz's 16-th summer, Weigel helped him begin to see the importance of _____ even in logic and philosophy.

One of the main differences in the foundations of Newton's calculus and that of Leibniz's was that Newton's machine had the _____ rule built in while Leibniz's machine had the _____ rule built into it.

Leibniz refered to his discovery of _____ as a "burst of light" and he was elected to the Royal Society in large part because of it. It greatly simplified some calculations for surface areas obtained by revolution and lead to a form of the fundamental theorem of calculus.

At Leibniz's request, the Royal Society formed a committee to determine who invented calculus. The committee found in favor of Newton, but this is not so surprising because _____ wrote the committee's report.

Bishop Berkely wrote: "And what are these _____? The velocities of evanescent increments. Ane what are these same evanescent increments? They are neigher finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them ghosts of departed quantities?"

Newton's book *Principia* is recognised as the greatest scientific book ever written. This is a full treatment of Newton's new physics and its application to _____.

_____, in *Tretise on fluxions*, gave the best response to Berkeley's well founded criticism of Newton's calculus, using the classical method of exhaustion in an attempt to bring rigorous logical arguments to the subject.

2. (*2+2+2+2=8 points*)

a. What is Newton's binomial coeficient theorem?

b. What is Newton's method (for finding/approximating roots)?

c. What did Newton mean by "Reversion of series?" What is it used for?

d. If $b_0 = 1$, why is it that

$$\frac{\sum_{i=0}^{\infty} a_i \, \mathbf{x}^{i+t}}{\sum_{i=0}^{\infty} b_i \, \mathbf{x}^i} = a_0 \, \mathbf{x}^t + (\text{terms of } \mathbf{x} \text{ of degree higher than t})?$$

3. (*8 points*)

Discuss the debate of credit for the invention of the Calculus between Newton and Leibniz. Be sure to include the roles played by Oldenberg, Keill and the Royal Society of London.

4. (*10 points*) (XIII, 7)

Using Newton's notation, if $\dot{y}/\dot{x} = cx^{n-1}\sqrt{a + bx^n + cx^{2n}}$, substitute $z = x^n$ in this manner to show that

$$y = \underline{\square}\frac{c}{n}\sqrt{a + bz + cz^2}$$

5. (*10 points*) (IX, 9)

Consider the parabola $y = \sqrt{x}, 0 \leq x \leq a$. Knowing that $D\sqrt{x} = 1/2\sqrt{x}$, show that the normal to the parabola is $n = \frac{1}{2}\sqrt{4x + 1}$. Use Leibniz's characteristic triangle to find that the area of the paraboid obtained by revolving this parabola about the x-axis, which is

$$A = \int 2\pi \, y \, ds$$