

**Math 406, Practice for Exam III**  
University of Illinois, December 2004

**Instructions:** Please answer the questions as clearly as possible and ask if you are unsure about what is needed for full credit in your solution. No computers, cell phones, calculators, books, cheat sheets, pets, acquaintances, friends, family, enemies, magic, psychic readings or chanting can be used to aid you with this exam. Good luck and remember that not all questions are weighted equally.

1. (12 points) The following are short answers and worth 2 points each.

The *Acta Eruditorum* published Leibniz's account of calculus before Newton's. It is not so surprising that this journal would do this because \_\_\_\_\_.

In 1705, Queen Anne \_\_\_\_\_ Newton, making him the first scientist to be so honoured for his scientific work.

\_\_\_\_\_, besides being a great mathematician, was also an important philosopher and is particularly well known for his theory of Monadology.

Leibniz received his masters degree in \_\_\_\_\_.

Despite publishing an influential philosophical paper in 1666 at the age of 20, Leibniz was denied a \_\_\_\_\_ and had to go to Altdorf to get it a year later.

Leibniz went to Paris in 1672 in order to \_\_\_\_\_.

After meeting Pell, Hooke and Boyle in England, Leibniz was unhappy to learn that \_\_\_\_\_ later spoke poorly of his mathematical accomplishments at the Royal Society.

Leibniz had promised the Royal Society that he would finish his \_\_\_\_\_ but made the society's president, Oldenburg, unhappy when he lost interest in it.

When Leibniz first wrote to the Royal Society about his investigations with infinitesimals, Oldenburg wrote back that Newton and Gregory had already found more \_\_\_\_\_.

When Tschirhaus joined Leibniz in \_\_\_\_\_ in 1675, Leibniz's calculus really began to make progress quickly.

Unable to stay in Paris, Leibniz went to Hanover to become a \_\_\_\_\_ which is where he remained until his death.

Keill wrote a paper in *Transactions of the Royal Society of London* accusing Leibniz of \_\_\_\_\_.

Despite rather unimpressive progress, Newton received his bachelors's degree in 1665. That summer the University was closed due to the \_\_\_\_\_ and Newton's scientific genius began to emerge when he had to return home.

Newton's original manuscript on calculus did not get published until after his death. Though originally written in Latin, it was first published in \_\_\_\_\_.

Newton's first published scientific paper was on \_\_\_\_\_ which was criticized by Hooke and led to several attempts by Newton to publically humiliate Hooke.

Newton retired from research in 1693 after having suffered a second \_\_\_\_\_.

Bishop \_\_\_\_\_, an eminent metaphysician, attacked the logical foundations of the calculus as developed by Newton.

Leibniz is one of the inventors of calculus but he is also called

“The first great German \_\_\_\_\_.”

One of the main differences in the foundations of Newton’s calculus and that of Leibniz’s was that Newton’s machine had the \_\_\_\_\_ rule built in while Leibniz’s machine had the \_\_\_\_\_ rule built into it.

Newton’s calculus was based upon what he called the “method of \_\_\_\_\_.”

\_\_\_\_\_ received his first degree at the age of 14 with a thesis that developed Newton’s theories when they were still relatively unknown. He met Newton on his first trip to England, later wrote a two volume work defending Newton’s calculus and was even partly supported by Newton himself at his position as Edinburgh where he remained until his death.

Newton discovered the power series for  $e^x$  by the method of \_\_\_\_\_ of power series.

Newton used what we now call \_\_\_\_\_ to accomplish what he called the reversion of series.

**2.** (2+2+2+4=8 points)

- a. What is Newton’s binomial coefficient theorem?
- b. What is Newton’s method (for finding/approximating roots)?
- c. What did Newton mean by “Reversion of series?” What is it used for?
- d. If  $b_0 = 1$ , why is it that

$$\frac{\sum_{i=0}^{\infty} a_i \mathbf{x}^{i+t}}{\sum_{i=0}^{\infty} b_i \mathbf{x}^i} = a_0 \mathbf{x}^t + (\text{terms of } \mathbf{x} \text{ of degree higher than } t)?$$

**3a.** (8 points)

Discuss how Newton arrived at his calculus. Be sure to include his binomial series, his form of the chain rule and fundamental theorem and revision of series.

**3b.** (8 points)

Discuss how Leibniz arrived at his calculus. Be sure to include his characteristic triangle and transmutation.

**4.** (10 points)

Using Newton’s notation, if  $\dot{y}/\dot{x} = \frac{b}{2\sqrt{x}}\sqrt{a+b\sqrt{x}}$ , substitute  $z = \sqrt{x}$  in this manner to show that

$$y = \square b\sqrt{a+bz}$$

**5.** (10 points)

Consider the parabola  $y = \sqrt{1-x^2}, 0 \leq x \leq 1$ . Knowing that  $D\sqrt{x} = -x/\sqrt{1-x^2}$ , show that the normal to the parabola is  $n = 1$ . Use Leibniz’s characteristic triangle to find that the area obtained by revolving this about the  $x$ -axis, which is

$$A = \int 2\pi y ds$$

Use a graph of the curve and simple geometry to justify your answer.