Math 415 Solutions for 7/15

§ 5.2.

\# 13. (a) $C_1 = 0$, $C_2 = -1$, $C_3 = 0$, $C_4 = 1$.

(b) By observation, $C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} \longrightarrow C_2$

by cofactor of row 1 and cofactor of column 1. $C_4 = -1 \cdot C_2$

Easy to generalize for any $n \geq 4$, $C_n = -1 \cdot C_{n-2}$.

Thus $C_{10} = -C_8 = C_6 = -C_4 = -1$.

\#19. Refer to P_{535} of the textbook for explanation.

\#23. (a) One way to prove is to use Laplace expansion by first n columns ($n=2$ here)

(b) Not unique.

(c) Not unique. A theorem claims that $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \det(AD - CB)$ if $AC = CA$.

(I've found some online resources for your reference :

an article about determinants of block matrix:

$\boxed{\text{http://www.mth.kcl.ac.uk/~jrs/gazette/blocks.pdf.}}$

§ 5.3.

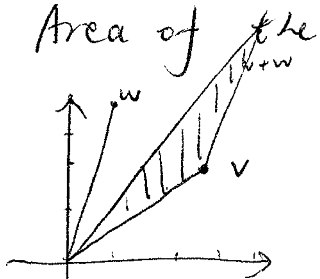
\# 1a. By Cramer's rule,

$$x_1 = \frac{\begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix}} = -2. \qquad x_2 = \frac{\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix}} = 1$$

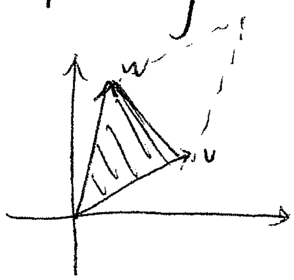
\# 2b. $y = \dfrac{fg - id}{D}$. where D is the 3 by 3 coeffecient determinant.

\# 16. (a). Area of the parallelogram $= \left| v, w \right| = \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 10$.

(b). Area of the triangle $= \dfrac{\text{abs}}{2}\left(\begin{vmatrix} 3, & 2, & 1 \\ 1, & 4, & 1 \\ 4, & 6, & 1 \end{vmatrix} \right) = 5.$



(c). Area of the triangle $= 5.$



\#21. Refer to P_{536} of the textbook.

\#23. As above.

\#27. $\underline{1 \;\&\; r}$. $\underline{J = r}$. ($J = L_1 \cdot L_2$ since two columns are orthogonal)

\#29. $J^{-1} = \begin{vmatrix} \partial r/\partial x & \partial r/\partial y \\ \partial\theta/\partial x & \partial\theta/\partial y \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\frac{1}{r}\sin\theta & \frac{1}{r}\cos\theta \end{vmatrix}$