

Quiz 10, Math. 415,

Friday, July 24th, 2009

Explain your answers carefully. Write complete sentences, not just formulas.

1 (15 points) Consider the curve in the x - y -plane, with equation

$$f(x, y) = \mathbf{x}^t A \mathbf{x} = 1, \quad A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Is the curve an ellipse or an hyperbola? Explain you answer.

2. (15 points) Calculate the singular values of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

3.a (10 points) Explain when two $n \times n$ matrices A, B are *similar*.

3.b (10 points) Show that $\det(A) = \det(B)$ if A, B are similar.

3.c (10 points) If A, B are similar, and A has eigenvector \mathbf{x} and eigenvalue λ , find an eigenvector and eigenvalue for B . Explain.

- 4 You know that a rank one matrix A can always be written as $A = \mathbf{xy}^t$ for some column vectors \mathbf{x}, \mathbf{y} .
- a. (15 points) If $A = \mathbf{xy}^t$ is an $m \times n$ matrix of rank one, what are the sizes of \mathbf{x} and \mathbf{y} ? So the questions is to find p, q such that $\mathbf{x} \in \mathbb{R}^p, \mathbf{y} \in \mathbb{R}^q$.

- b. (15 points) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, find \mathbf{x}, \mathbf{y} such that $A = \mathbf{xy}^t$.