

Quiz 11, Math. 415,

Friday, July 31st, 2009

Explain your answers carefully. Write complete sentences, not just formulas.

1 (15 points) Let a linear transformation $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be given by

$$L \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad L \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}.$$

Find $L \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

2. (15 points) Consider the basis $F = \left\{ \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right\}$. Find the coordinate vector of $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ with respect to the basis F .

- 3** Let $L = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x_1 - x_2 \\ -x_1 + x_2 \end{pmatrix}$. This is a linear transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$. (You don't need to show this.)
- a.** (15 points) Find the matrix of L , with respect to the standard basis.

- b.** (15 points) Consider the basis $F = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ of \mathbb{R}^2 . Find the matrix of the linear transformation L given above using the basis F both in the input and in the output space. (This is the matrix L_{FF} in the notation of the handout.)

- 4 (15 points) Consider the vector space P^2 of polynomials of degree 2 or less, and the basis $F = \{x^2, 1, x\}$. (Note the order!). Give the coordinate vector of $f(x) = 3 + 4x + x^2$ with respect to this basis F .
5. (15 points) Consider the second derivative linear transformation $D^2: P^3 \rightarrow P^1$, given by $D^2(f(x)) = \frac{d^2 f}{dx^2}$. Find the matrix of D^2 using the bases $\{1, x, x^2, x^3\}$ and $\{1, x\}$.