



- 4 Consider the Markov matrix  $A = \begin{pmatrix} 0.8 & 0.05 \\ 0.2 & 0.95 \end{pmatrix}$  and the discrete dynamivcal system  $u_{k+1} = Au_k$  for  $u_k, u_{k+1}$  two component column vectors. So  $u_1 = Au_0, u_2 = Au_1 = A^2u_0$  and more generally  $u_k = A^k u_0$ .
- a. (15 points) Find the eigenvalues of  $A$ , if the eigenvectors are given as  $\mathbf{x}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$  and  $\mathbf{x}_2 = \begin{pmatrix} 0.2 \\ 0.8 \end{pmatrix}$ .

- b. (15 points) If  $u_0 = \mathbf{x}_1 + \mathbf{x}_2$ , find  $u_k$  and find the limit  $u_\infty = \lim_{k \rightarrow \infty} u_k$ . Explain.

- 5** Consider the  $3 \times 3$  Fourier matrix  $F = \begin{pmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w^4 \end{pmatrix}$ , where  $w$  is the basic 3rd root of 1:  $w = e^{2\pi i/3}$
- a.** (15 points). Find the length of the second column of  $F$ .

- b.** (15 points) Find the complex dot product(inner product) of the second and third columns of  $F$ .