

Quiz 3, Math. 415,

Wednesday, June 24th, 2009

Explain your answers carefully. Write complete sentences, not just formulas.

1.a (10 points) Consider the permutation matrix

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

What is the inverse of P ?

1.b (10 points) Let \mathbf{x}, \mathbf{y} be column vectors (of the same length). Express the dot product $\mathbf{x} \cdot \mathbf{y}$ in terms of the transpose.

1.c (10 points) Explain why $P\mathbf{x} \cdot P\mathbf{y} = \mathbf{x} \cdot \mathbf{y}$ if P is a permutation matrix.

2. Consider subsets $S \subset \mathbb{R}^3$. Which of these is actually a *subspace*? Explain!

a. (10 points) S is the set of vectors $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, with $b_1 = 1$.

b. (10 points) S is the set of vectors $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, with $b_3 = 0$.

c. (10 points) S is the set of vectors $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, with $b_1^2 + b_2^2 + b_3^2 = 1$.

3. Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$.

a. (15 points) Define the column space $C(A)$ and calculate it for the above matrix A . The column space is a subspace of \mathbb{R}^n for what n in this case? What is the *shape* of $C(A)$?

b. (15 points) Define the null space $N(A)$ and calculate it for the above matrix A . The null space is a subspace of \mathbb{R}^n for what n in this case? What is the *shape* of $N(A)$?