

Quiz 4, Math. 415,

Friday, June 26th, 2009

Explain your answers carefully. Write complete sentences, not just formulas.

1 Consider the matrix $A = \begin{pmatrix} 1 & 2 & 10 \\ 2 & 4 & 20 \\ 4 & 8 & 40 \end{pmatrix}$.

1.a (10 points) What is the rank of A ?

1.b (10 points) How many free variables has A ?

1.c (10 points) Find all solutions of $Az = 0$.

Still $A = \begin{pmatrix} 1 & 2 & 10 \\ 2 & 4 & 20 \\ 4 & 8 & 40 \end{pmatrix}$.

1.d (15 points) Find the condition(s), if any, on $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, so that the system $A\mathbf{x} = \mathbf{b}$ is solvable.

e. (15 points) If $\mathbf{b} = \begin{pmatrix} -3 \\ -12 \\ -24 \end{pmatrix}$, is $A\mathbf{x} = \mathbf{b}$ solvable? If so find all solutions, otherwise explain.

2. Let A be a 3×4 matrix (3 rows, 4 columns).
- a. (10 points) Can the rank of A be 4? Explain!

- b. (20 points) If the rank of a 3×4 matrix is 3, what can you say about the system $A\mathbf{x} = \mathbf{b}$? Is it solvable for all \mathbf{b} ? If it is solvable for a given \mathbf{b} , is the solution unique?