**Quiz 6, Math. 415,** **Wednesday, July 8th, 2009**

Explain your answers carefully. Write complete sentences, not just formulas.

**1** (15 points) Let $P^2$ be the vector space of all polynomials of degree 2 or less and let $V \subset P^2$ be the subspace of polynomials $f(x)$ with $f(0) = 0$. (This is a subspace, but you don't need to prove this.)

Find a basis for $V$ and its dimension .

**2** (15 points) Let $\mathtt{Mat}_{3\times 3}$ be the vector space of $3 \times 3$ matrices, and let $\mathcal{U}$ be the subspace of *upper triangular* matrices: elements of $\mathcal{U}$ have the form $\begin{pmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{pmatrix}$.

Find a basis for $\mathcal{U}$ and its dimension.

**3** (15 points) Consider the *incidence* matrix $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$. Draw the network corresponing to $A$. Carefully label the edges and nodes, and indicate the direction of the edges.

**4** (15 points) Consider the matrix $A = \begin{pmatrix} 2 & -2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ and the vector $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. You know that $\mathbf{x}$ can be wrtitten as $\mathbf{x} = \mathbf{x}_r + \mathbf{x}_n$ where $\mathbf{x}_r$ is in the row space of $A$ and $\mathbf{x}_n$ in the null space. Find $\mathbf{x}_r$ and $\mathbf{x}_n$.

**5** Let $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

**a.** (15 points) Find the projection $\mathbf{p}$ of $\mathbf{b}$ on the line $L$ through $\mathbf{a}$.

**b.** (15 points) If $L$ is still the line through $\mathbf{a}$, what is the point on $L$ that is closest to $\mathbf{b}$?