

Quiz 7, Math. 415,

Friday, July 10th, 2009

Explain your answers carefully. Write complete sentences, not just formulas.

- 1 (15 points) Let $\mathbf{q}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{q}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $V = \text{Span}(\mathbf{q}_1, \mathbf{q}_2)$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$. Find the closest point on V to \mathbf{b} . (Hint: what is $\mathbf{q}_1 \cdot \mathbf{q}_2$?)

2. (15 points) Let $A = (\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n)$ be an $m \times n$ matrix such that $A^t A$ is a diagonal $n \times n$ matrix:

$$A^t A = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & d_n \end{pmatrix}.$$

What can you say about the columns of A ?

3 (30 points) Use the Gram-Schmidt process to find an orthonormal basis for \mathbb{R}^2 , starting with $a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, a_2 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$. Find the QR decomposition of $A = (a_1 \ a_2)$.

- 4 (30 points) A scientist tries to find the relation between the mysterious quantities x and y . She measures the following values:

x		0		1		2		-3
y		3		2		1		0

Set up, but *do not solve*, the equations to find the best straight line through these points.