

**Quiz 9, Math. 415,**

**Wednesday, July 22nd, 2009**

Explain your answers carefully. Write complete sentences, not just formulas.

1 (15 points) Solve

$$\frac{d\mathbf{u}}{dt} = A\mathbf{u}, \quad u(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad \text{with } A = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

2. (15 points) Calculate  $\exp(tA)$  if  $A = \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}$ .

3. (15 points) Consider the differential equation  $y'' + 4y' + 3y = 0$ . Rewrite this equation in vector form:

$$\frac{d\mathbf{u}}{dt} = A\mathbf{u}.$$

What is  $A$ ? (You don't have to solve the system.)

4. (15 points) Consider the *quadratic form*  $f(x, y, z) = 4x^2 - 2xy + 3y^2 + 4xz + 6z^2 + 8yz$ . Find a matrix  $A$  such that

$$f(x, y, z) = \mathbf{x}^t A \mathbf{x}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

**5** (15 points) Let  $f(x, y) = \mathbf{x}^t A \mathbf{x}$  with  $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ ,  $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ . Analyze the critical point of  $f(x, y)$  at  $(0, 0)$ . Is it a maximum, minimum or other? Explain!

**6** (15 points) Explain what a positive definite matrix is, and decide whether or not  $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$  is positive definite. Explain!