Solution for Quiz 7. Math 415

1. Note that $\vec{q}_1 \cdot \vec{q}_2 = 0$, $\vec{q}_1$ and $\vec{q}_2$ are orthogonal.

The projection of $\vec{b}$ onto $\vec{q}_1$: $\dfrac{\vec{b} \cdot \vec{q}_1}{\|\vec{q}_1\|^2} \cdot \vec{q}_1 = \dfrac{7}{3} \vec{q}_1$.

The projection of $\vec{b}$ onto $\vec{q}_2$: $\dfrac{\vec{b} \cdot \vec{q}_2}{\|\vec{q}_2\|^2} \cdot \vec{q}_2 = \dfrac{-1}{2} \vec{q}_2$.

The closest point on $V$ to $\vec{b}$ is $\dfrac{7}{3} \vec{q}_1 - \dfrac{1}{2} \vec{q}_2$.

2. The columns of $A$ are orthogonal, $\|a_i\|^2 = d_i$.

3. $\hat{a}_1 = \dfrac{a_1}{\|a_1\|} = \dfrac{\sqrt{5}}{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\xrightarrow{\text{Gram-Schmidt}}$

$$a_2 - \frac{a_2 \cdot a_1}{\|a_1\|^2} a_1 = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \frac{6}{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -\frac{6}{5} \\ \frac{3}{5} \end{pmatrix}$$

$$\hat{a}_2 = \frac{\sqrt{5}}{3}\begin{pmatrix} -\frac{6}{5} \\ \frac{3}{5} \end{pmatrix} = \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$$

QR Decomposition of $A$: $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} \sqrt{5} & \frac{6}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{pmatrix}$

4. We assume the relation between $x$ and $y$ is linear (that's why we need to find the best straight line). $y = ax + b$. we want to find $a, b$.

The data yields $A \cdot \begin{pmatrix} a \\ b \end{pmatrix} = p$, where $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ -3 & 1 \end{pmatrix}$,

$p = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} a \\ b \end{pmatrix} = (A^T A)^{-1} (A^T p)$.