Quiz 8, Math. 415, **Friday, July 17th, 2009**

Explain your answers carefully. Write complete sentences, not just formulas.

1 Let A, B be 3×3 matrices, with $\mathrm{Det}(A) = 5, \mathrm{Det}(B) = 4$.

a. (5 points) Calculate $\mathrm{Det}(2A)$.

$$\mathrm{Det}(2A) = 2^3 \,\mathrm{Det}\, A = 8 \times 5 = 40$$

b. (5 points) Calculate $\mathrm{Det}(A^{-1})$.

$$\mathrm{Det}(A^{-1}) = \mathrm{Det}(A)^{-1} = \frac{1}{5}$$

c. (5 points) Calculate $\mathrm{Det}(A^t B)$.

$$\mathrm{Det}(A^t B) = \mathrm{Det}\, A^t \,\mathrm{Det}\, B = \mathrm{Det}\, A \,\mathrm{Det}\, B$$

$$= 5 \times 4 = 20$$

2. (5 points) Calculate $\operatorname{Det}\begin{pmatrix} 3 & 3 & 4 \\ 6 & 8 & 7 \\ -3 & 5 & -9 \end{pmatrix}$.

$$\begin{vmatrix} 3 & 3 & 4 \\ 6 & 8 & 7 \\ -3 & 5 & -9 \end{vmatrix} = \begin{vmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 8 & -5 \end{vmatrix} = \begin{vmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{vmatrix}$$

$$= -6$$

3. (5 points) Calculate $\operatorname{Det}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= 1$$

4 (5 points) Find the area of the triangle with vertices $(0,0)$, $(1,3)$, $(5,1)$.

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 1 \\ 5 & 1 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 3 \\ 5 & 1 \end{vmatrix} = \frac{1}{2}(1\times 1 - 5\times 3)$$

$$= -7$$

5 (7 points) Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Is A diagonalizable? If so, give the diagonalization, otherwise explain.

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 3 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 = 0$$

So $\lambda = 1$ is the eigenvalue of A.

$$\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0.$$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. There is only one eigenvector with $\lambda=1$.

A ~~can't be~~ is not diagonalizable.

6 (8 points) Solve

$$\frac{d\mathbf{u}}{dt} = A\mathbf{u}, \quad u(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad \text{for } A = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) = 0$$

$\lambda_1 = 1 \qquad \lambda_2 = 3.$

$\lambda_1 = 1$: $\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. The eigenvector with $\lambda_1 = 1$ is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\lambda_2 = 3$: $\begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_2\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$. The eigenvector with $\lambda_2 = 3$ is $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$.

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \frac{3}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}.$$

$$u(t) = \frac{3}{2}e^{t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3e^{3t}\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{2}e^{t} + \frac{3}{2}e^{3t} \\ 3e^{3t} \end{pmatrix}$$