Quiz 1 Solutions.

1. Sol: $2\vec{v} = \begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix} \Rightarrow \vec{v} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$

$2\vec{w} = \begin{pmatrix} 6 \\ -6 \\ 2 \end{pmatrix} \Rightarrow \vec{w} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$

2. Sol: $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+z \\ y+z \\ z \end{pmatrix}$

So $Ax = \begin{pmatrix} 12 \\ 9 \\ 5 \end{pmatrix}$

2a. Sol: $\vec{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2^2+1^2}} = \begin{pmatrix} \frac{2}{5}\sqrt{5} \\ -\frac{\sqrt{5}}{5} \end{pmatrix}$

2b. Sol: $\vec{v} \cdot \vec{w} = 0 \Rightarrow 2W_1 - W_2 = 0 \Rightarrow W_2 = 2W_1$

thus $\vec{w} = \begin{pmatrix} a \\ 2a \end{pmatrix}$ for $a \in \mathbb{R}$.

3a. Sol: Let $A = (W_1, W_2, W_3)$. i.e. $A = \begin{pmatrix} 1 & 4 & -2 \\ 2 & 5 & -1 \\ 3 & 6 & 0 \end{pmatrix}$

Solve $Ax = 0$.

$$A \longrightarrow \begin{pmatrix} 1 & 4 & -2 \\ 0 & -3 & 3 \\ 0 & -6 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 4 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

thus $x = \begin{pmatrix} -2x_3 \\ x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \cdot x_3$ for $\forall x_3 \in \mathbb{R}$. $\begin{cases} x_1 = -2a \\ x_2 = a \\ x_3 = a. \end{cases}$ $\forall a \in \mathbb{R}$.

3b. Sol: Since by (3a) $-2\vec{w}_1 + \vec{w}_2 + \vec{w}_3 = 0$

$\vec{w}_1, \vec{w}_2, \vec{w}_3$ are not linear independent $\Rightarrow$ Not 3 dim

$\vec{w}_1, \vec{w}_2$ are independent $\Rightarrow$ 2 dim.

Thus the combinations of the three vectors form a plane.