Quiz 2, Math. 415, **Friday, June 19th, 2009**

Explain your answers carefully. Write complete sentences, not just formulas.

1. (15 points) Consider a 2×2 system $A\mathbf{x} = \mathbf{b}$ (two equations in two variables). Explain why it is not possible that the system has *exactly* two solutions.

Write $B = (A, b)$. The solution of $Ax = b$ has only two possibilities:

1). If rank A = rank B, then $Ax = b$ has 1 solution or infinitely many solutions.

2) If rank $A <$ rank B, then $Ax = b$ has no solution.

Or one can read P 159 of the text book.

2. (15 points) Find the 3×3 matrix E such that

$$E \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 3x + z \end{pmatrix}.$$

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

3a. (15 points) Find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ or explain A is not invertible.

Since $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

the rank of A is 1. Hence A is not invertible.

3b. (15 points) Find the inverse of $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ or explain A is not invertible.

Comput A^{-1} by Gauss - Jordan Elimination:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 6 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right).$$

Hence $A^{-1} = \begin{pmatrix} 1 & -2 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$.

4a. (15 points) Find the LU decomposition of $A = \begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix}$.

$$\begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix} \xrightarrow{-2\cdot \text{row1} + \text{row2}} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Hence $A = LU = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

4b. (15 points) Solve the system $A\mathbf{x} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$ for the above $A = \begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix}$.

$$Ax = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

$$Ax = LUx = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

Let $c = Ux$. Then,

$$Lc = \begin{pmatrix} 5 \\ 11 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} c = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

$$c = \begin{pmatrix} 5 \\ 1 \end{pmatrix}.$$

$$Ux = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$