

Quiz 3. Solutions

1. a.
$$P^T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

1. b.
$$\vec{x} \cdot \vec{y} = x^T y.$$

1. c.
$$Px \cdot Py = (Px)^T Py = x^T P^T P y = x^T y = x \cdot y.$$

2. a. No.

Reason 1: No zero vector.

Reason 2: Not addition/scalar closed.

b. Yes. Straightforward to verify it satisfies the definition of subspace.

c. No. Reason: No zero vector.

3. a. Define: the column space $C(A)$ is the collection of linear combinations of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$.

$$C(A) = \left\{ a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} : \forall a \in \mathbb{R} \right\}.$$

$$n = 3.$$

$C(A)$ represents a line in \mathbb{R}^3 .

b. Define: the null space $N(A)$ is the solution space of $Ax = 0$

$$n = 3.$$

$N(A)$ is two-dimensional (standing for a plane (perpendicular to the line of $C(A)$) in \mathbb{R}^3).