Quiz 5 Solutions

1. Sol: Write $A = [a_1, a_2, a_3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 7 & 10 \end{bmatrix}$

Reduce A to the elementary form, or alternatively note that $a_1 + a_2 = a_3$.

$\{a_1, a_2, a_3\}$ isn't a basis.

2. Sol: We know $\{1, x, x^2\}$ is a basis for P^2.

Hence $\{f_1, f_2\}$ can't be a basis for P^2.

3. Sol: Suppose E_{ij} denotes a Matrix whose i-th row, j-th column entry is 1 and all other entries are zero.

then $\{E_{ij}\}_{\substack{1 \le i \le 3 \\ 1 \le j \le 2}}$ is a basis for $Mat_{3\times 2}$.

Dimension is $2 \times 3 = 6$.

4. Sol: the plane: $2x + 2y - z = 0$

the normal vector is $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

thus write up a perpendicular vector of $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

eg. $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ ~~and using Gram-Schmidt metho~~

and another which is independent of $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

eg. $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$ forms a basis.

5. Sol:

a. $A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \end{pmatrix}$

$Ax = 0$ implies $x = \begin{pmatrix} -3x_2 - 4x_3 \\ x_2 \\ x_3 \end{pmatrix}$

thus $N(A) = \left\{ \begin{pmatrix} -3a - 4b \\ a \\ b \end{pmatrix} : \forall a, b \in \mathbb{R} \right\}$.

b. $C(A) = \left\{ a \begin{pmatrix} 1 \\ 3 \end{pmatrix} : a \in \mathbb{R} \right\}$.

c. Consider $A^T x = 0$. we get $x = \begin{pmatrix} -3x_2 \\ x_2 \end{pmatrix}$

thus $N(A^T) = \left\{ a \begin{pmatrix} -3 \\ 1 \end{pmatrix} : a \in \mathbb{R} \right\}$.

d. $C(A^T) = \left\{ a \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} : a \in \mathbb{R} \right\}$.