Exam I, Math 501–review
February 25, 2008
8:30–9:50

The exam has 5 questions (some with multiple parts). The exam begins:

With the exception of problems 1 and 3a, you are allowed to use results from class, the notes or the homework. If you are uncertain if your solution is complete or may not be demonstrating what is intended please ask. Each problem is worth 10 points (equally weighted) so be sure to do those questions which you are most confident about success first.

1. This question is intended to test ones knowledge of the axioms for M to be an R-module. A sample problem of this type would be homework I.5.

2. This question is intended to test ones knowledge of (co)limits of R-modules by using a fairly concrete and small diagram. An example of a question like this would be:

Given the following diagram of R-modules and R-module homomorphisms

$$\begin{array}{ccc} C & \xrightarrow{\alpha} & A \\ \big\downarrow{\scriptstyle \beta} & & \\ B & & \end{array}$$

prove that the colimit of the diagram is isomorphic to $Z = A \oplus B/\sim$ (where $(\alpha(c), 0) \sim (0, \beta(c))$ for all $c \in C$) with structure maps $A \to Z$ ($a \mapsto \overline{(a, 0)}$) and $B \to Z$ ($b \mapsto \overline{(0, b)}$).

3. This problem is intended to test ones knowledge of the difference between a ring map and a module map using group rings as examples. An example of question like this would be (not the group ring case):

Let p be a prime. Prove that $Hom_{\text{Rings}}(\mathbb{Z}/p, \mathbb{Z}/p^2)$ is empty but that $Hom_{\mathbb{Z}}(\mathbb{Z}/p, \mathbb{Z}/p^2) \cong \mathbb{Z}/p$. Or, that $Hom_{\text{Rings}}(\mathbb{Z}[x], \mathbb{Z}) \cong \mathbb{Z}$ but $Hom_{\mathbb{Z}}(\mathbb{Z}[x], \mathbb{Z}) \cong Hom_{\text{Sets}}(\mathbb{N}, \mathbb{Z})$.

4. This problem is intended to test ones knowledge about the Invariant Basis Property. In particular, to show that an explicit ring has the IBP while another explicit example does not. An example question like this would be:

Let R be a commutative ring. Show that $R[\mathbb{Z}]$ satisfies IBP but that $Hom_R(R[\mathbb{Z}], R[\mathbb{Z}])$ does not.

5. This problem is intended to test ones knowledge of Hom_R and its relations to (possibly) infinite sums and products of modules. The two homework problems are good examples of this, I.8 and II.4