

Exam II, Math 501
April 14, 2008
8:30-9:50

The exam has 6 questions (with multiple parts). It begins:

If you are uncertain if your solution is complete or may not be demonstrating what is intended please ask. Each problem is worth 10 points (equally weighted) so be sure to do those questions which you are most confident about success first.

1. This problem asks for several explicit examples and short justifications over topics like simple, semi-simple, Noetherian, Artinian, injective, projective, flat, tensor products, etc. For example, give an example of a simple module of a ring which is not-semisimple. There are five in total, equally weighted.
2. This problem combines the ideas of colimits/limits with those of Artinian/Noetherian. For example, one can show that the pull-back of Artinian/ Noetherian modules is again Artinian/Noetherian.
3. This problem asks you to do an explicit example of determining the decomposition of a group algebra into a product of matrix rings of division algebras. This of course uses a combination of Maschke's Theorem, Schur's Lemma and Wedderburn's Theorem in theory, but in truth you'll really just need to determine $\mathbb{C}[G]$ for a specific example, and the group can't be too big. For example, if $|G| = 9 = 3^2$, we know the center is non-trivial (p -group) and so G has more than 3 conjugacy classes. If G is non-abelian we also know there are fewer than 9 conjugacy classes. So, $9 = 4 + 1 + 1+1+1+1$ is the only case and $\mathbb{C}[G] \cong \mathbb{C}^{\times 5} \times M_2(\mathbb{C})$ (with appropriate references to major theorems tossed in along the way).
4. Please don't get confused on this problem. It is about the coproduct of commutative k -algebras but it is stated in terms of group algebras (as is often done on the comp exams). The homework problems are fairly good examples of these. Something along these lines would be to show that $R[G] \otimes_R R[H] \cong R[G \times H]$ as rings if R is commutative and G and H are finite groups.
5. This problem has to do with injective/projective modules and maps splitting. For example, if B is projective R module then show that for every surjective module map $A \xrightarrow{p} B$ and R -module X , $\text{Hom}(B, X) \xrightarrow{p^*} \text{Hom}(A, X)$ is split injective.
6. This problem has to do with the classification of semi-simple rings. In particular, it uses some of the major results along these lines we developed in class as well as some of our homework on short exact sequences.