

Introduction to Differential Equations – Math 285 G1
Spring 2009
Quiz 1

1. Write down the general solution to $\frac{dy}{dt} = ky$.

Solution. We can actually solve this in several ways, but this is also something we'll see so much that it's worth knowing cold. If not, the easiest way to solve this is to note that it is separable:

$$\begin{aligned}\frac{dy}{y} &= k dt, \\ \int \frac{dy}{y} &= \int k dt, \\ \ln(y) &= kt + C, \\ y(t) &= e^{kt+C} = Ce^{kt}.\end{aligned}$$

To check that this is the solution, notice that

$$\frac{dy}{dt} = Cke^{kt} = ky.$$

2. Find the particular solution to $y' = 2y, y(0) = 4$.

Solution. From above, we know that the general solution to the ODE is

$$y(t) = Ce^{2t}.$$

Plugging in $t = 0$, we get

$$y(0) = Ce^0 = C \cdot 1 = C,$$

so $C = 4$, so the solution is

$$y(t) = 4e^{2t}.$$

3. Solve $y' + y = 2$.

Solution. One can solve this problem in two ways (we will later learn a third), and I'll do both.

- (a) First notice that this problem is linear, and we can try to solve it like that. This is in the form $y' + P(x)y = Q(x)$, and we have $P(x) = 1$. We know the integrating factor will be

$$\rho(x) = \exp\left(\int P(x) dx\right) = e^x,$$

so we multiply the equation by e^x throughout, giving

$$e^x \frac{dy}{dx} + e^x y = 2e^x.$$

Now, we notice that

$$\frac{d}{dx}(e^x y) = e^x \frac{dy}{dx} + e^x y$$

from the product rule, and so we have

$$\begin{aligned}\frac{d}{dx}(e^x y) &= 2e^x, \\ e^x y &= \int 2e^x dx = 2e^x + C, \\ y(x) &= 2 + Ce^{-x}.\end{aligned}$$

(b) We could also notice that the equation can be made separable, i.e. write it as

$$\frac{dy}{dx} = 2 - y,$$

and so we write

$$\begin{aligned}\frac{dy}{2-y} &= dx, \\ -\ln(2-y) &= x + C, \\ \ln(2-y) &= -x + C, \\ 2-y &= e^{-x+C} = Ce^{-x}, \\ y(t) &= 2 - Ce^{-x}.\end{aligned}$$