Partial Differential Equations – Math 442 C13/C14
Fall 2009
Homework 4 — due October 9

1. Consider the boundary value problem

$$A'' + \lambda A = 0, \quad A'(0) + aA(0) = 0, \quad A(L) = 0.$$

   (a) Show that if $a < 0$, then there is no negative eigenvalue.

   (b) Under which conditions is there a zero eigenvalue?

   (c) Show there are infinitely many positive eigenvalues for any value of a.

   Bonus: We showed in (a) that if $a < 0$ then there is no negative eigenvalue. If turns out that for some positive a, this problem has a negative eigenvalue (and for some others it does not). Write down a condition on a which determines whether such an eigenvalue exists.

2. **(Strauss 4.3.2.)** Consider the eigenvalue problem with Robin boundary conditions

$$A'' + \lambda A = 0, \quad A'(0) - \alpha_0 A(0) = 0, \quad A'(L) + \alpha_L A(L) = 0.$$

   (a) Show that zero is an eigenvalue if and only if $\alpha_0 + \alpha_L = -\alpha_0 \alpha_L L$.

   (b) Compute the eigenfunction corresponding to this eigenvalue.

3. Solve the equation

$$u_t = k u_{xx}, \quad x \in [0, \infty), \quad t > 0,$$
$$u(x,0) = \begin{cases} 1, & x \in (0,1), \\ 0, & x > 1, \end{cases}$$
$$u(0,t) = 0.$$

4. Consider the Schrödinger equation with Neumann boundary conditions:

$$i u_t = u_{xx}, \quad \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0.$$

   Write out the general series solution for this equation as we have done for the heat and wave equations, i.e. separate variables, get ODEs in x and t, solve these problems, and take the linear combination.