Methods of Mathematical Physics - 556 X1
Homework 5
Due December 1, 2008

1. (Problem 4.1.11 from Keener.) The convolution of regular functions is defined as

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x-t)g(t)\,dt.$$

   (a) Define the convolution of distributions.

   (b) What is $\delta * \delta$?

   (c) What is $\delta * H$, where H is the Heaviside distribution?

   (d) Can you define the convolution of a distribution and a smooth function?

2. Let $f(x)$ be the function such that $f(x) = x^2$ for $x \in [-1,1]$, and f is extended periodically, i.e. for any $x \notin [-1,1]$, there is a unique k so that $x - 2k \in [-1,1]$, so define $f(x) = f(x-2k)$. Then compute f', f'' in a distributional sense.

3. (Problem 4.2.1 from Keener.) Construct the Green's function and express the solution in terms of the Green's function:

$$u'' = f(x), \quad u(0) = 0, \quad u'(1) = 0.$$

4. (Problem 4.2.2 from Keener.) Construct the Green's function and express the solution in terms of the Green's function:

$$u'' = f(x), \quad u(0) = 0, \quad u'(1) = u(1).$$

5. (Problem 4.3.1 from Keener.) Let L be defined as $Lu = u'' + a(x)u' + b(x)u$, with boundary condition $u(0) = u'(1)$, $u(1) = u'(0)$. Compute L^* with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx.$$

   (Don't forget to compute the domain of L^* in this and the following questions!)

6. (Problem 4.3.2 from Keener.) Let L be defined as $Lu = -(p(x)u')' + q(x)u$, with boundary condition $u(0) = u(1)$, $u'(0) = u'(1)$. Compute L^*.

7. Consider the linear operator $Lu = u''$ where we define the separable boundary conditions

$$\alpha u(0) + \beta u'(0) = 0,$$
$$\gamma u(1) + \delta u'(1) = 0,$$

   for some real numbers $\alpha, \beta, \gamma, \delta$. Compute L^*. What needs to be true for L to be self-adjoint?