

December 2, 2005

Name .....

EXAM 3 *100pts.*  
MATH 118 SECTION X1

1 *20 pts.* Consider the weighted voting system [4: 1, 2, 1, 1, 1]. ( Let voter A have 1 vote, B have 2 votes, C, D and E have 1 vote each.)

(a) What is the total number of pivotal votes (that all the voters together) will have? Why?

Ans: There are 5 voters in all and there are  $5! = 120$  ways of ordering them. So there are 120 pivotal votes.

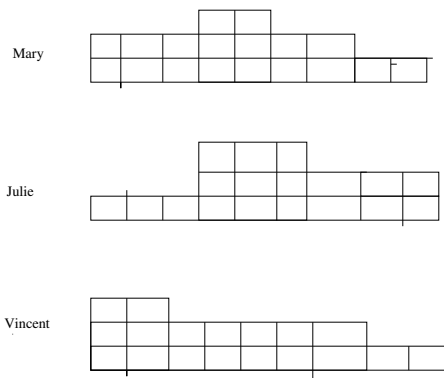
(b) How many pivotal votes does A have? Show work.

Ans: Following are different ways A can be pivotal. A can be in the fourth position in which case B should follow A. There are  $3! = 6$  ways of this happening. A can be in the third position, in which case B should be before A. If A is in third and B in first position, there are  $3! = 6$  possibilities. If A is in third position and B is in second position there are again 6 ways of this happening. So in all A has 18 pivotal votes.

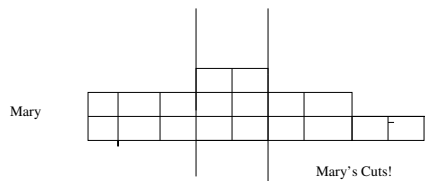
(c) How many pivotal votes does B have? Show work.

Ans: We found that A has 18 votes, so will C, D and E. That makes it a total of 72 pivotal votes. All the other votes will come from B. So B will have  $120 - 72 = 48$  pivotal votes.

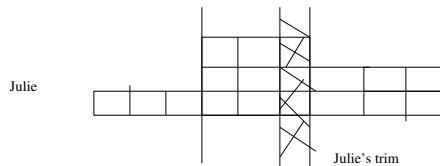
2 24 pts. Mary, Julie and Vincent want to divide fudge cake brownie with nuts. The nuts are not evenly distributed and Mary has a preference for the crusty edges. We would like to use Selfridge Conway method to distribute it amongst the three of them with the order being Mary, Julie and Vincent. Following are their preferences when they distribute 18 square units over parts of the brownie.



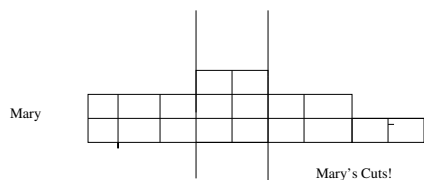
Show work for all your answers!!!  
 (a) Mary cuts the brownie into what she thinks are three equal pieces. Show how she will cut if only vertical cuts are allowed.



Ans:  
 (b) Julie gets to trim the largest piece ( as she sees it ) down to the second largest piece. This is so that according to her now there are two equally large pieces. Show how she would trim it.



Ans:  
 2(c) Which piece will Vincent pick? How much are the new pieces (ignoring the trimming ) worth to him? Why?  
 Ans: Vincent will pick the first piece. That is because the first one is worth 8 units, the second is worth 4 and the trimmed piece is worth 4 units for him.



2(d) Which pieces do Mary and Julie end up with? Explain your answer.  
 Ans: Julie has to take the trimmed piece (the last one) since Vincent did not take it. That leaves Mary with the middle piece.

3 *20 pts.* Miranda and Neil are planning to share an apartment. They decide to resolve the possible issues that might arise using the Adjusted Winner Procedure. As per the method they both assign points totaling 100 on various issues as follows.

	Miranda	Neil
Stereo Level	10	20
Cleanliness	25	20
Phone Time	25	10
Room Party Policy	10	15
Visitor Policy	20	10
Alcohol Use	10	25

Write down who will get a say in what issue. Show work.

Ans: Just from points we see that Miranda will get a say in Cleanliness, Phone time and Visitor Policy. Neil will get a say in Stereo level, Room Party Policy, and Alcohol use. But this gives Neil only 60 points compared to 70 points that Miranda will get. Then Miranda ought to share some issue with Neil.

To decide which issue should be shared, compute the ratio of issues that Miranda is getting in comparison to Neil's values for it.

$$\text{Cleanliness} = 25/20 = 1.25$$

$$\text{Phone time} = 25/10 = 2.5$$

$$\text{Visitor Policy} = 20/10 = 2$$

So Miranda should share her say in Cleanliness with Neil. If Miranda gets  $x$  of the share then Neil gets  $1 - x$  share of the say. Then for them to have same points in the end we need

$$45 + 25x = 60 + 20(1 - x)$$

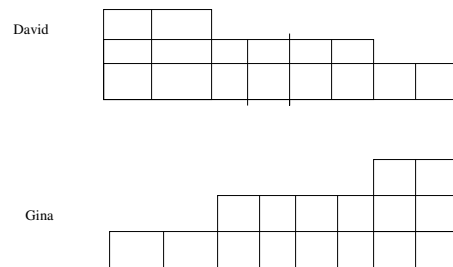
$$45 + 25x = 80 - 20x$$

$$45x = 80 - 45 = 35$$

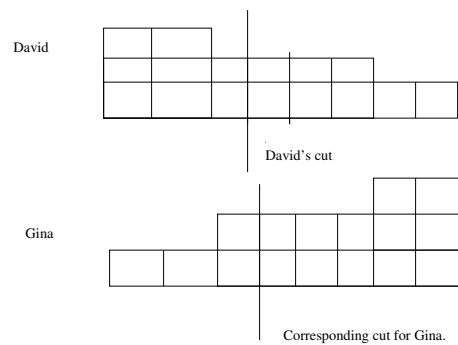
$$x = 35/45 = 7/9$$

Thus Miranda gets a  $\frac{7}{9}$ th share say in Cleanliness , Phone time and Visitor Policy. Neil will get a say in Stereo level, Room Party Policy, Alcohol use and  $\frac{2}{9}$ th share say in Cleanliness.

- 4 *15pts.* David and Gina want to divide their garden. David wants to plant vegetables whereas Gina wants to plant flowers. When allowed to divide 16 square to describe their preferences here is what they write out. All the cuts made are vertical with respect to their preferences.

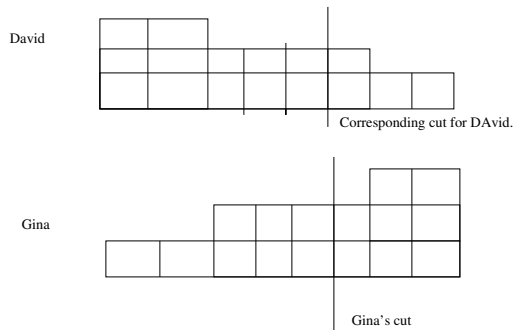


- (i) Let David divide and Gina choose. Where is the cut made and who gets which piece?



Ans: Gina will choose the second piece and Davide gets the first piece.

- (ii) Let Gina divide and David choose. Where is the cut made and who gets which piece?



Ans: David will choose the first piece and Gina will get the second piece.

- (iii) Looking at your answers above, which is better; to divide or to get to choose? Does it matter? Explain your answer.

Ans: In the above cases the person who chooses seems to get more than half of the land in their value. Whereas the one who divides gets exactly what is half in their value.

5 *21pts.* The following are short-answer questions. Show work for your answers.

- (a) Natalie, Bob and Peter inherit their grandmother's antique dining table. They use Knaster's Inheritance procedure to decide who gets the table. If Natalie bids 1050 dollars, Bob bids 900 and Peter bids 1200 then who gets the table? How much money do the others get? Show work.

Ans: Peter gets the table since he bid the highest. Peter thinks he should give both Natalie and Bob  $1200/3 = 400$  each. But Natalie only expects  $1050/3 = 350$  and Bob expect  $900/3 = 300$ . After giving them what they expect Peter is still left with  $800 - 650 = 150$  dollars which he had expected to give away. So now divide  $150/3 = 50$ . Peter then gives both Natalie and Bob 50 dollars more. In the end Peter gets the table and has to give away 750 dollars. Natalie gets 400 dollars and Bob gets 350.

- (b) Is the Last Diminisher method for cake cutting proportional? Why or Why not? Explain your answer.

Ans: Yes. Last Diminisher the person cutting the piece to what he thinks is a fair share gets the piece if no one else trims it. In which case no one else thought it was more than a fair share. If anyone trims it then they will be trimming it to what they think is a fair share for them. The person who trims the last gets to keep the piece and in

that case the person does think its a fair share.

- 5(c) Neil, Bob and Miranda use the Lone Divider method to divide a piece of land. Neil divides the land into three plots. Both Bob and Miranda want the same piece of land. What are they supposed to do next? How do they decide who gets which piece? Explain your answer.

Ans: If Bob and Miranda both want the same piece they give the piece they do not want to Neil. Neil leaves the game. Bob and Miranda then put back the other two pieces together and use divide and choose to decide their shares.