May 9, 2004

MATH 225 N2
Supplementary Exercises

These exercises are only supplementary to classwork and all previous homeworks and review sheet. They do not encompass everything taught in class

- 1.1 8, 9, 12, 15, 17, 27.
- 1.2 11, 12, 15, 16, 18, 19
- 1.3 11, 17, 19, 21, 22.
- 1.4 11, 14, 17, 18, 19, 21, 31, 32.
- 1.5 5, 23 c, 24c,e
- 1.7 9, 11, 13, 21 a, c, d.
- 2.1 9, 20, 21, 23
- 2.2 13, 14, 21
- 2.3 5, 11, 12a, b, c.
- 2.6 6
- 3.1 7, 11, 23
- 3.2 9, 17, 19, 26, 33
- 3.3 3, 13
- 4.1 13, 17
- 4.2 11, 25a, b , f, 26a, b, c, 29.
- 4.3 5, 7, 11, 13, 21a, b, c, d.
- 4.5 3, 7, 13, 15.
- 4.6 1, 3, 7, 9, 11.
- 5.1 26, 27

- 5.2 9, 13, 25.
- 5.3 17, 23, 26.
- 6.1 3, 5, 11, 13, 19 d, 23, 27.
- 6.2 3, 7, 9, 13, 17, 24 a , d, e, 29.
- 6.3 1, 3, 7, 11, 15, 21 a, d.
- 6.5 1, 9, 13, 17 a, c.
- 6.6 1

(1) Consider the linear system whose augmented matrix is of the form

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{bmatrix}$$

For what values of a will the system have a unique solution?

(2) Give a general solution to the following system of equations.

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & 4 \\ x_1 & - & 2x_2 & + & 2x_3 & = & 1 \\ 11x_1 & + & 2x_2 & + & x_3 & = & 14 \end{array}$$

(3) Let A and B be $n \times n$ matrices and let $C = AB$. Prove that if B is not invertible then C is not invertible.

(4) Let $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$. Solve the matrix equation $AX + B = C$ where X is a 2×2.

(5) Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

(6) Find all possible values of d that would make the following matrix invertible.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 9 & d \\ 1 & d & 3 \end{bmatrix}$$

(7) Determine whether the following spaces are subspaces.

– $\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} / x_1 x_2 = 0 \right\}$ of $\mathbb{R}^2$

– $\left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} /\text{ a, b, c are real numbers} \right\}$ of 2×2 real matrices.

(8) In each of the following, determine the dimension of the subspace spanned by the given vectors and give its geometric description.

(i) $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

(9) Let A be a $m \times n$ matrix and B is an invertible $m \times m$ matrix. Show that BA and A have the same null space and hence the same rank.

(10) If A is a $m \times n$ matrix of rank r, what are the dimensions of $N(A)$ and $N(A^T)$.

(11) Let S be a subspace of R^3 spanned by $x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Find a basis of the orthogonal complement of S and give a geometrical description of the same.

(12) Find the best least squares fit by a linear equation ($y = mx + c$) to the data

x	y
-1	0
0	1
1	3
2	9

(13) Write short answers to the following.

(i) Let $\begin{bmatrix} 1 & 3 & -1 \\ 0 & -5 & 2 \\ 2 & -1 & 0 \end{bmatrix}$ be the inverse of A . Find an appropriate matrix so that $AX = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$. Is X invertible? Why or why not?

(ii) A is a diagonalizable 2×2 matrix with eigenvalues 1 and -1. Show that $A^2 = I$.

(iii) If A is a $n \times n$ matrix such that $AA^T = I$ then, what values will $\det A$ take?

(iv) If $\{v_1, v_2, v_3\}$ are linearly independent vectors in $I\!R^5$ and $v_4 = v_3 - v_2 + v_1$, then is $\{v_1, v_2, v_4\}$ is linearly independent? Why or why not?

(v) If A has eigenvalues 1, 3 and $\frac{2}{3}$, find determinant of A.

(vi) If A is a 3×3 invertible matrix and v_1, v_2, v_3 are linearly indpendent vectors in $I\!R^3$. Show that Av_1, Av_2, Av_3 are linearly independent.

14 State True or False with justification.(*No points for just stating true or false*)

(i) Let $C = AB$ for 4×4 matrices A and B. If C is invertible then A is invertible.

(ii) Let W be a subspace of $I\!R^4$ and v be a vector in $I\!R^4$. If $v \in W$ and $v \in W^\perp$ then $v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

(iii) Let V be a vector space and W be a subspace of V. If $dimW = dimV$ then $W = V$.

(iv) If A is a invertible 3×3 matrix and B and C are 3×3 matrices , then $AB = AC$ implies $B = C$.