

Problem Set #2, Math 347 (Rezk D1).

Friday, January 29, 2010.

Note: in the first three problems, you may use any fact about *fields* without comment (i.e., ordinary algebraic manipulations). However, any fact about the *order* properties of an ordered field must be proved using the axioms for an ordered field; you may also use statements about ordered fields which I prove in class using the ordered field axioms.

1. Let S be an ordered field, and suppose $x, y, u, v \in S$. Using only the Ordered Field axioms, show that $x \leq y$ and $u \leq v$ imply $x + u \leq y + v$.
2. Let S be an ordered field and suppose $x \in S$. Using only the Ordered Field axioms, show that $0 < x$ implies $0 < x^{-1}$.
3. Let S be an ordered field and suppose $x, y \in S$. Using only the Ordered Field axioms, show that $0 < x < y$ implies $0 < y^{-1} < x^{-1}$.
4. Problems from MT: 1.33, 1.34, 1.36.