Math 461
HW1 Solns
Due 9/4/09

1. The area of a disk of radius r is πr^2 and the area of the rectangle is $4 \cdot 6 = 24$. Assuming the Monte Carlo model, we'd have

$$\frac{\pi r^2}{24} \approx \frac{1423}{10000}, \text{ or } r \approx \sqrt{\frac{(.1423)(24)}{\pi}} \approx 1.04$$

(ok answer)

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2. $A = \{2,4,6,8\}$ $\quad A^c = \{3,5,7\}$

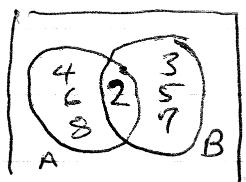
$B = \{2,3,5,7\}$ $\quad B^c = \{4,6,8\}$

$A \cup B = \{2,3,4,5,6,7,8\}$

$A \cap B = \{2\}$

$A^c \cap B = \{3,5,7\}$

$A \cup B^c = \{2,4,6,8\}$



$p(2) = \frac{1}{16}$, $p(3) = \frac{2}{16}$, $p(4) = \frac{3}{16}$, $p(5) = \frac{4}{16}$, $p(6) = \frac{3}{16}$, $p(7) = \frac{2}{16}$, $p(8) = \frac{1}{16}$

by counting the number of occurrences

Thus (a) $p(A) = \frac{1+3+3+1}{16} = \underline{\frac{8}{16}}$ (b) $p(B) = \frac{1+2+4+2}{16} = \underline{\frac{9}{16}}$

(c) $P(A \cup B) = \frac{1+2+3+4+3+2+1}{16} = \underline{1}$ (d) $p(A \cap B) = \underline{\frac{1}{16}}$ (or $= P(A) + P(B) - P(A \cup B)$)

(e) $p(A^c \cap B) = \frac{2+4+2}{16} = \frac{8}{16} = \frac{1}{2}$ (f) $p(A \cup B^c) = p(A) = \frac{8}{16} = \frac{1}{2}$

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3. The twelve outcomes are $(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)$. [I didn't specify ordered or unordered, so 6 outcomes: $\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$ is ok too.] The sums are $\{3,4,5,3,5,6,4,5,7,5,6,8\}$ and the probabilities change: $p(2) = \frac{0}{12}$ $p(3) = \frac{2}{12}$, $p(4) = \frac{2}{12}$, $p(5) = \frac{4}{12}$, $p(6) = \frac{2}{12}$, $p(7) = \frac{2}{12}$, $p(8) = \frac{0}{12}$, or: $p(2) = p(8) = 0$, $p(3) = p(4) = p(6) = p(7) = \frac{1}{6}$, $p(5) = \frac{1}{3}$. The sets A, B and their Boolean funcs do not change

(a) $p(A) = \frac{0+1+1+0}{6} = \frac{1}{3}$ (b) $p(B) = \frac{0+1+2+1}{6} = \frac{4}{6} = \frac{2}{3}$

(c) $p(A \cup B) = \frac{0+1+1+2+1+1+0}{6} = 1$ (d) $p(A \cap B) = \frac{0}{6} = 0$, (e) $p(A^c \cap B)$ $\frac{1+2+1}{6} = \frac{4}{6} = \frac{2}{3}$

(f) $p(A \cup B^c) = p(A) = \frac{1}{3}$. What happens is the $\begin{smallmatrix}11\\22\\33\\44\end{smallmatrix}$ are gone from the set.

4. $\Omega = \left\{ \begin{matrix} HHH, & THH \\ HHT, & THT \\ HTH, & TTH \\ HTT, & TTT \end{matrix} \right\}$. Translating from words,

$A = \left\{ \begin{matrix} HHH \\ HHT \\ TTH \\ TTT \end{matrix} \right\}$ $B = \left\{ \begin{matrix} HHT \\ HTT \\ THH \\ TTH \end{matrix} \right\}$ $C = \left\{ \begin{matrix} HHT \\ HTH \\ THT \\ TTH \end{matrix} \right\}$

$A \cap B = \left\{ \begin{matrix} HHT \\ TTH \end{matrix} \right\}$ $A \cap C = \left\{ \begin{matrix} HHT \\ TTH \end{matrix} \right\}$ $B \cap C = \left\{ \begin{matrix} HHT \\ TTH \end{matrix} \right\}$

$A \cap B \cap C = \left\{ \begin{matrix} HHT \\ TTH \end{matrix} \right\}$, so

$P(A) = P(B) = P(C) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = P(A \cap C) = P(B \cap C) = \frac{2}{8} = \frac{1}{4} = P(A \cap B \cap C)$

The significance of this example is that any two of the three events A, B, C are independent, but they are not mutually independent 3 at a time

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Bonus space filler.

Venn Diagram for 4 Events