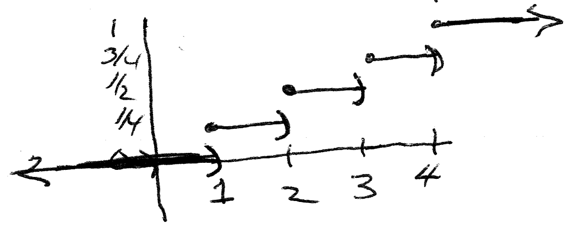
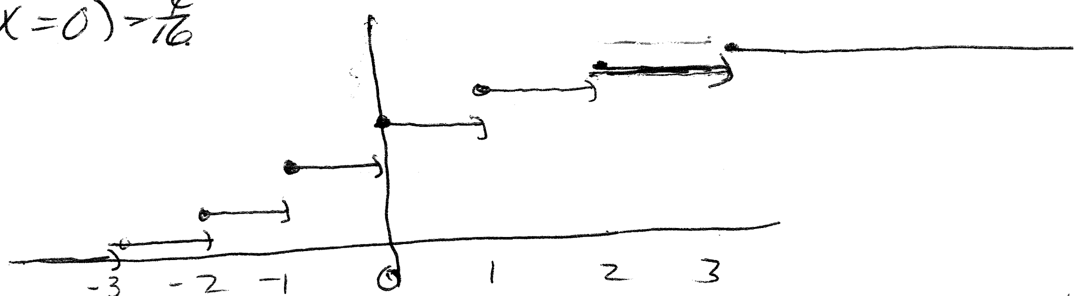
Math 461
HW 2
Solutions
Due 9/11/02

1a) Without repeating the diagram, the frequencies for X are: $P(X=1)=\frac{4}{16}=\frac{1}{4}=P(X=2)=P(X=3)=P(X=4)$

The cdf is $P(X\le c)$
Basically, you count the # of $\{1,2,3,4\}$ which are $\le c$



1b. From the table, reading across, $X-Y$ takes the values $\{0,1,2,3,-1,0,1,2,-2,-1,0,1,-3,-2,-1,0\}$, so
$P(X=-3)=\frac{1}{16}=P(X=3)$, $P(X=-2)=\frac{2}{16}=P(X=2)$, $P(X=-1)=\frac{3}{16}=P(X=1)$
and $P(X=0)=\frac{4}{16}$



The jumps have size $\frac{1}{16},\frac{2}{16},\frac{3}{16},\frac{4}{16},\frac{3}{16},\frac{2}{16},\frac{1}{16}$ in turn. I don't expect you to draw any better than me

1c. $E(X)=\frac{1+2+3+4}{4}=\underline{\underline{\frac{5}{2}}}$ $\quad Var(X)=E(X^2)-(E(X))^2$ by formula
$=\frac{1^2+2^2+3^2+4^2}{4}-\left(\frac{5}{2}\right)^2=\frac{1+4+9+16}{4}-\frac{25}{4}=\frac{30}{4}-\frac{25}{4}=\underline{5/4}$

Or $=E((X-\frac{5}{2})^2)=\frac{(1-\frac{5}{2})^2+(2-\frac{5}{2})^2+(3-\frac{5}{2})^2+(4-\frac{5}{2})^2}{4}=\frac{\frac{9}{4}+\frac{1}{4}+\frac{1}{4}+\frac{9}{4}}{4}=\frac{20}{16}=\frac{5}{4}$

1d. $E(X-Y)=0$ I'll accept "by observation", though you can also do it directly $\frac{0+1+\cdots+-1+0}{16}=0$

$Var(X-Y)=E((X-Y)^2)-(E(X-Y))^2=E((X-Y)^2)-0=E((X-Y)^2)$
$=\frac{0+1+4+9+1+0+1+4+4+1+0+1+9+4+1+0}{16}=\frac{40}{16}=\underline{\underline{\frac{5}{2}}}$

We'll see later that X and Y are independent and so
so $Var(X-Y)=Var(X)+(-1)^2 Var(Y)=\frac{5}{4}+\frac{5}{4}=\frac{5}{2}$

2a. A slightly different data set

Get $P(X=1) = \frac{3}{12} = \frac{1}{4} = P(X=2) = P(X=3) = P(X=4)$

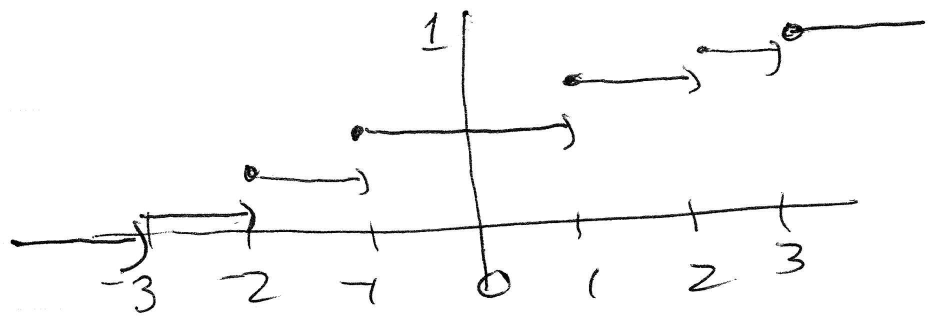
so the frequencies are the same as 1a, hence so is the cdf

2b. $X-Y$ takes the values $\{-1,1,2,3,-2,-1,1,2,-3,-2,-1,1\}$

which is 1b, but with the 0's missing

$P(X=-3) = P(X=3) = \frac{1}{12}$ $P(X=-2) = P(X=2) = \frac{2}{12}$ $P(X=-1) = P(X=1) = \frac{3}{12}$

The cdf is now flat at $x=0$; the jumps have size $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{3}{12}, \frac{2}{12}, \frac{1}{12}$



2c. Again, $E(X) = \frac{1+2+3+4}{4} = \frac{5}{2}$ and $Var(X) = \frac{5}{4}$, because it has the same distribution as in 1c.

2d. Again $E(X-Y)=0$ by "symmetry" and $E((X-Y)^2) = Var(X-Y)$

It's: $\frac{1+1+4+9+4+1+1+4+9+4+1+1}{12} = \frac{40}{12} = \frac{10}{3}$. This is larger than before, "because" you have killed all the data points near the mean, so the spread has to be larger.

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3. Consider $P(X \le t)$. If $t < -2$, then $P(X \le t) = 0$

If $t \ge 3$, then $P(X \le t) = 1$. If $-2 \le t < 3$, then $P(X \le t)$ is

the probability that if you pick a point in $[-2,3]$ it's in $[-2,t]$

which is $\frac{t-(-2)}{3-(-2)} = \frac{t+2}{5}$

(Continuity means probability at any single point is 0.)