Math 461
HW5
Recap

1. Lots of problems here, some with the numbers. If you still have questions after seeing the solutions, please ask in class. There was one clever alternate solution. Let $P(M) = a$. We won't calculate a, but remove it from the algebra. I don't endorse this, but it worked.

$$\frac{7}{10} = P(M \mid Y) = \frac{P(M \cap Y)}{P(Y)} = \frac{P(Y \mid M) P(M)}{P(Y)} = \frac{\frac{2}{3} \cdot a}{p} \qquad (1)$$

$$\frac{2}{10} = P(M \mid Y^c) = \frac{P(M \cap Y^c)}{P(Y^c)} = \frac{P(Y^c \mid M) P(M)}{P(Y^c)} = \frac{\frac{1}{3} \cdot a}{1-p} \qquad (2)$$

(Note: $P(Y^c \mid M) = 1 - P(Y \mid M)$ because if M happens, either Y or Y^c occurs).

Now take $\frac{(1)}{(2)}$. $\frac{7}{2} = \frac{\frac{2}{3} a}{p} \Big/ \frac{\frac{1}{3} a}{1-p} = \frac{\frac{2}{3}}{\frac{1}{3}} \cdot \frac{1-p}{p} = \frac{2-2p}{p}$

So $7p = 4 - 4p \Rightarrow 4 = 11p \Rightarrow p = \frac{4}{11}$.

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2. Two things: (i) On the actual probabilities, a statement such as $f_I(p) = 2p^2$ is impossible on its face, because if $p^2 > \frac{1}{2}$, then the probability of success is > 1. What happens here is that you forget to subtract the overcounting of both components working $f_I(p) = 2p^2 - p^4$!
(ii) $f_I(p) - f_{II}(p) = -p + 2p^2 - p^3$. I can accept: (a) This is $-p(1-p)^2 < 0$ for $0 < p < 1$ so it's negative or (b) A graph for $0 \le p \le 1$ showing it's negative. These are what mathematicians do and what engineers do, respectively. I cannot accept "It's negative" or "I put in one or two values and it's negative". That's not good enough and I think you know it!

You need to be reflective about your answers!