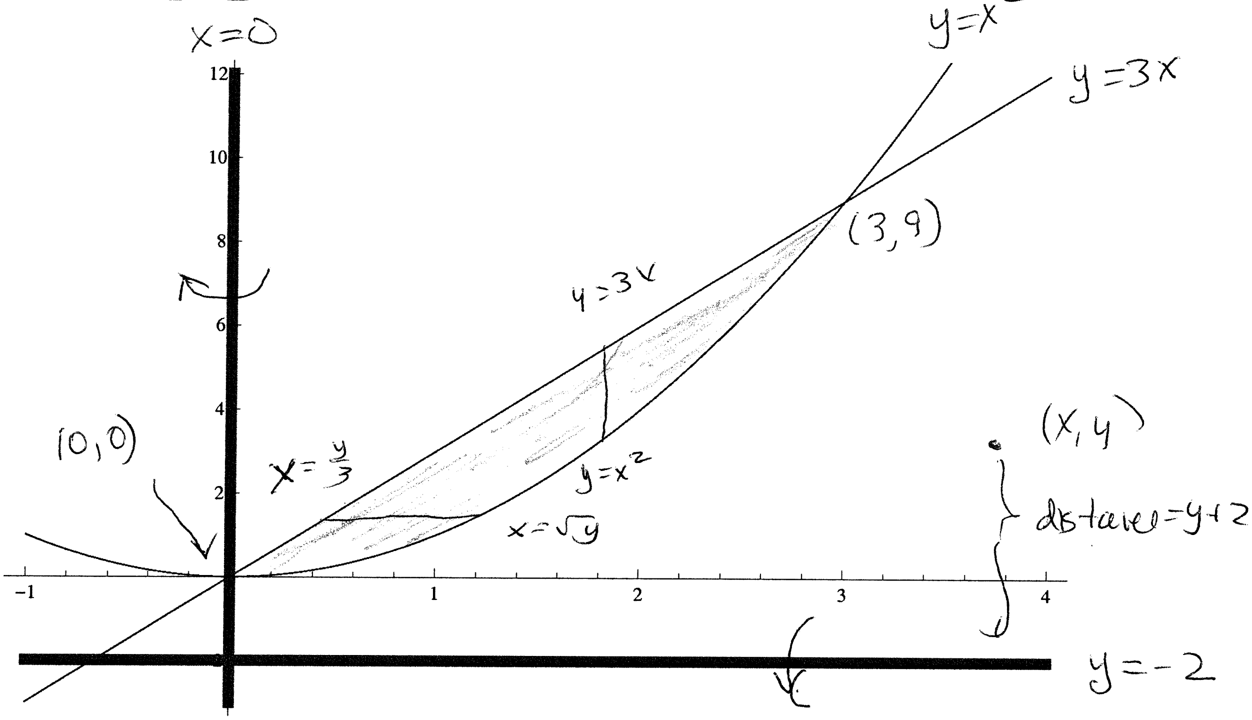
Last Quiz Answers



R is the region between $y=3x$ and $y=x^2$.
The intersections are where $3x=x^2 \Rightarrow x=0$ and $x=3$
and $x^2 \le 3x$ for $0 \le x \le 3$.
In terms of vertical strips,

$$R=\{(x,y):\ 0\le x\le 3,\ x^2\le y\le 3x\}$$

In terms of horizontal strips

$$R=\{(x,y):\ 0\le y\le 9,\ \frac{y}{3}\le x\le \sqrt{y}\}$$

I'll do 1 and 2 both as shells and as washers.

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1. Rotate R around y-axis

Shells: $\displaystyle\int_{x=0}^{3} 2\pi x(3x-x^2)\,dx = \int_0^3 (6\pi x^2-2\pi x^3)\,dx$

$$= 6\pi\cdot\frac{x^3}{3} - 2\pi\cdot\frac{x^4}{4}\Big|_0^3 = \left(6\pi\cdot\frac{27}{3}-2\pi\cdot\frac{81}{4}\right)-(0-0)$$

$$= 54\pi - \frac{81\pi}{2} = \boxed{\frac{27\pi}{2}}$$

Washers $\displaystyle\int_{y=0}^{9} \pi\left((\sqrt{y})^2-\left(\frac{y}{3}\right)^2\right)dy = \int_0^9 \pi\left(y-\frac{1}{9}y^2\right)dy$

$$= \pi\cdot\frac{y^2}{2} - \pi\cdot\frac{y^3}{27}\Big|_{y=0}^{9} = \pi\cdot\frac{81}{2}-\pi\cdot\frac{729}{27} = \boxed{\frac{27\pi}{2}}$$

2. Rotate R about $y=-2$. Recall that this is two units below the x-axis, and this shifts the radius of rotation up by 2.

Washers: $\displaystyle\int_{x=0}^{3} \pi\{(3x+2)^2 - (x^2+2)^2\}\,dx$

$$= \int_0^3 \pi(9x^2+12x+\cancel{4} - x^4 - 4x^2 - \cancel{4})\,dx = \pi\left(3x^3+6x^2 - \frac{x^5}{5} - \frac{4}{3}x^3\right)\Big|_0^3$$

$$= \pi\left(81+54 - \frac{243}{5} - \frac{4}{3}\cdot 27\right) - 0 = \boxed{\frac{252\pi}{5}}$$

Shells $\displaystyle\int_{y=0}^{9} 2\pi(y+2)\left(\sqrt{y} - \frac{y}{3}\right)dy$

$$= 2\pi\int_0^9 \left(y^{3/2} - \frac{y^2}{3} + 2y^{1/2} - \frac{2y}{3}\right)dy = 2\pi\cdot\left(\frac{2}{5}y^{5/2} - \frac{y^3}{9} + \frac{4}{3}y^{3/2} - \frac{y^2}{3}\right)\Big|_0^9$$

$$= 2\pi\left(\frac{2}{5}9^{5/2} - \frac{9^3}{9} + \frac{4}{3}\cdot 9^{3/2} - \frac{9^2}{3}\right) = 2\pi\left(\frac{486}{5} - 81 + 36 - 27\right) = 2\pi\cdot\frac{126}{5} = \boxed{\frac{252}{5}\pi}$$

Two important notes: (1) you only have to solve a problem once, not twice (2) The algebra is more complicated than you'll find on a quiz or exam, but the concepts are reasonable

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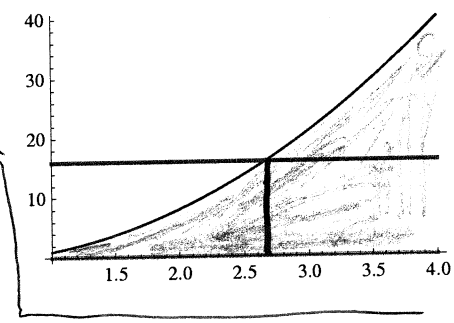
3. The average value of $f(x)$ on $[1,4]$ is defined to be

$$\frac{1}{4-1}\int_1^4 f(x)\,dx = \frac{1}{3}\int_1^4 (3x^2-2x)\,dx = \frac{1}{3}\cdot(x^3-x^2)\Big|_1^4 = \frac{1}{3}\left[(4^3-4^2)-(1^3-1^2)\right]$$

$= \frac{1}{3}(64-16) = \frac{48}{3} = 16$. If $f(c)=16$, then $3x^2-2x-16=0$

or $x = \dfrac{2\pm\sqrt{4-4\cdot 3(-16)}}{6} = \dfrac{2\pm\sqrt{196}}{6} = \dfrac{2\pm 14}{6} = -2, \boxed{\frac{8}{3}}$



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4. If $y = \frac{2}{3}x^{3/2}$, then $\frac{dy}{dx} = \frac{3}{2}\cdot\frac{2}{3}x^{1/2} = x^{1/2}$, and the desired arclength is

$$\int_{x=0}^{2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^2 \sqrt{1+(x^{1/2})^2}\,dx = \int_{x=0}^{2}(1+x)^{1/2}\,dx$$

Let $u = 1+x$, $du = dx$.

When $x=0$, $u=1$
When $x=2$, $u=3$

Arclength is $\displaystyle\int_{u=1}^{3} u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_1^3 = \frac{2}{3}(3\sqrt{3}-1) = \boxed{2\sqrt{3} - \frac{2}{3}}$