Math 461 Notes 8/26/09

Probability is a branch of mathematics which studies models predicting behavior

A probabilistic model consists of outcomes x contained in a universe Ω. We are usually interested in events A, which contain various outcomes and are subsets of the universe. Each event A has a probability $P(A)$ which is a real number in $[0,1]$. The main rules about probability are that if A and B are disjoint events (ie, $A \cap B = \emptyset$) then $p(A \cup B) = p(A) + p(B)$, and that $p(\Omega) = 1$

The spirit of this course is examples, rather than definitions, so here are a few.

1. Ω is a finite set $\{x_1, .., x_n\}$ and you model picking an element "at random" from the set. Here, if $A \subseteq \{x_1, .., x_n\}$, and $|A|$ denotes the number of elements in A, then $p(A) = \frac{1}{n} \cdot |A|$.

Special cases of this include

(i) $\Omega = \{1, .., 6\}$ and you roll a die

(ii) Flipping a "fair" coin n times. Coins can come up H or T (heads or tails). We make assumption that each of the 2^n tuples like HTH....TT is equally likely.

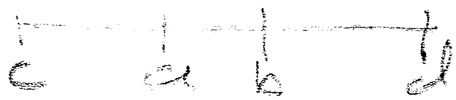
Eg $n=3$ $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Is it true that each is equally likely. I don't know. This is our model

2. Ω is a geometric object and $\mathcal{A}$ consists of (some) subsets of Ω. We have what experts will call a measure on Ω. (This gets very complicated.) The probability of choosing A is related to its size relative to Ω. Special cases

(i) $\Omega = [0,1]$. You model picking a point at random in Ω. In this case, if $A = [a,b]$, then $p(A) = \frac{b-a}{1-0}$

If $\Omega = [c,d]$ is a general interval in $\mathbb{R}$



$$p(A) = \frac{b-a}{d-c}$$

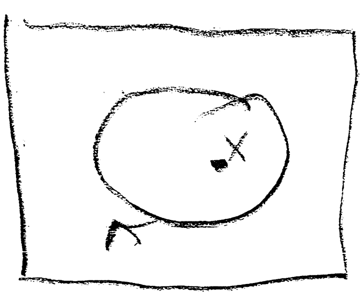
(ii) Ω = plane region $\quad p(A) = \frac{\text{area}(A)}{\text{area}(\Omega)}$

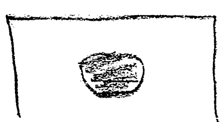
Note, in each case, the probability of picking a single point is zero. There are paradoxes if you look too carefully at this.

Venn-isms (John Venn 1834-1923)

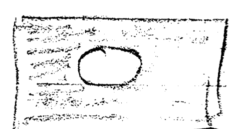
Lots of people understand things visually.

Ω





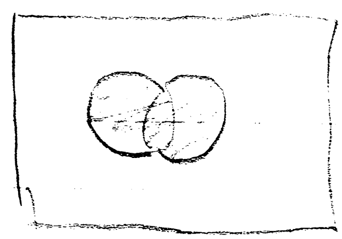
A



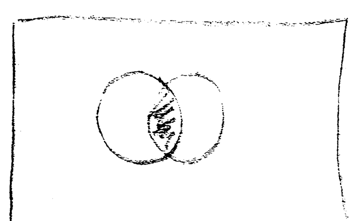
A^c or $\bar{A}$, the complement of A

This depends on Ω!

A shaded ≡
B shaded |||



$A \cup B$



$A \cap B$

Coming soon: $(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

Visually clear!