1.

	City	Burbs	Downstate
yes on A	.24	.1225	.125
No on A	.16	.2275	.125

Math 461
HW 4 soln's
9/25/09

A = "yes on A" C = city B = burbs D = downstate

$P(C) = .4$ $P(B) = .35$ $P(D) = .25$
$P(A|C) = .6$, $P(A|B) = .35$, $P(A|D) = .5$ } data given

so $P(A|C) = \frac{P(A\cap C)}{P(C)} \Rightarrow .6 = \frac{P(A\cap C)}{.4} \Rightarrow P(A\cap C) = .24$

it follows that $P(A\cap C^c) = P(A) - P(A\cap C) = .4 - .24 = .16$
(or $P(A^c|C) = 1 - P(A|C)$)

Similarly $P(A\cap B) = P(A|B)P(B) = (.35)(.35) = .1225$
and $P(A^c\cap B) = .35 - .1225 = (.65)(.35) = .2275$
and $P(A\cap D) = P(A|D)P(D) = (.5)(.25) = .125$
so $P(A^c\cap D) = P(D) - P(A\cap D) = .25 - .125 = .125$

Using those values, which can be calculated in several ways:

(a) $P(A) = P(A\cap C) + P(A\cap B) + P(A\cap D) = .24 + .1225 + .125 = .4875$

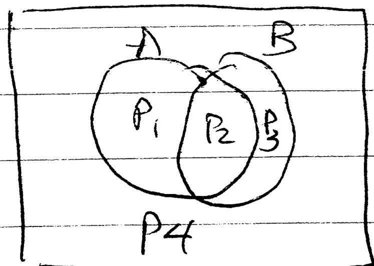
(b) $P(C|A) = \frac{P(A\cap C)}{P(A)} = \frac{.24}{.24+.1225+.125} = \frac{.24}{.4875} \simeq .492$

(c) $P(C|A^c) = \frac{P(A^c\cap C)}{P(A^c)} = \frac{.16}{.16+.2275+.125} = \frac{.16}{.5125} \simeq .312$

So .. 49% of the Prop A supporters live in the city and 31% of the Prop A opponents live in the city.
Overall, 40% of all voters live in the city

2\. Let $A = \{\text{component 1 works}\}$
$B = \{\text{compont 2 works}\}$ parallel circuit so only one conpent needs to work

Given $P(A) = .9$, $P(B) = .8$, $P(A \cup B) = .96$



← A guaranteed way to do this!

$$.9 = P(A) = P_1 + P_2$$
$$.8 = P(B) = P_2 + P_3$$
$$.96 = P(A \cup B) = P_1 + P_2 + P_3$$

$P_1 = P(A \cap B^c)$, $P_2 = P(A \cap B)$, $P_3 = P(A^c \cap B)$, $P_4 = P(A^c \cap B^c)$

By subtracting repeatedly, we see that $P_1 = .16$, $P_3 = .06$ and $P_2 = .74$. Since $1 = P_1 + P_2 + P_3 + P_4$, $P_4 = .04$

(Also $\underbrace{P(A \cap B)}_{= P_2} = P(A) + P(B) - P(A \cup B) = .9 + .8 - .96 = .74$

There are several paths to this

Now to the questions:

(a) $P(B^c | A^c) = \dfrac{P(B^c \cap A^c)}{P(A^c)} = \dfrac{P_4}{P_3 + P_4} = \dfrac{.04}{.06 + .04} = \dfrac{4}{10} = .4$

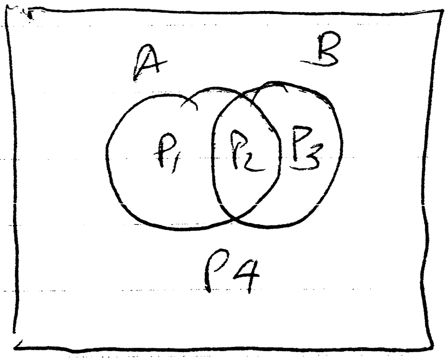
(b) This is $P(A \cap B) = P_2 = .74$

Not asked:
If A and B were independent, then we'd have

$P_2 = P(A)P(B) = (.9)(.8) = .72$ (not .74)
$P_1 = P(A)P(B^c) = (.9)(.2) = .18$ (not .16)
$P_3 = P(A^c)P(B) = (.1)(.8) = .08$ (not .06)
$P_4 = P(A^c)P(B^c) = (.1)(.2) = .02$ (not .04)

So $P(A \cup B) = .98$ (not .96) $\quad P(B^c | A^c) = P(B^c) = .2$ not .4
$P(A \cap B) = .72$ (not .74) $\quad$ is much smaller.

3. Welcome to the wonderful world of storyless algebra problems



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P_2}{P_2+P_3} = \frac{1}{3}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P_2}{P_2+P_1} = \frac{1}{4}$$

$$P(A^c \cap B^c) = P_4 = \frac{1}{5}$$

So $\frac{P_2}{P_2+P_3} = \frac{1}{3} \Rightarrow 3P_2 = P_2 + P_3 \Rightarrow P_3 = 2P_2$

$\frac{P_2}{P_2+P_1} = \frac{1}{4} \Rightarrow 4P_2 = P_2 + P_1 \Rightarrow P_1 = 3P_2$

$1 = P_1 + P_2 + P_3 + P_4 = 3P_2 + P_2 + 2P_2 + \frac{1}{5} \Rightarrow \frac{4}{5} = 6P_2$

$\Rightarrow P_2 = \frac{4}{30} = \frac{2}{15}$, $P_1 = \frac{6}{15}$, $P_3 = \frac{4}{15}$, $P_4 = \frac{1}{5} = \frac{3}{15}$

$P(A) = P_1 + P_2 = \frac{8}{15}$ $P(B) = P_2 + P_3 = \frac{6}{15}$ $P(A \cap B) = P_2 = \frac{2}{15}$

$P(A \cup B) = P_1 + P_2 + P_3 = \frac{12}{15} = 1 - \frac{1}{5}$

My story: A = "The Cubs win"
B = "I watch the game"

2) A = "I locked my car" ....
B = "I can't remember whether I locked my car"