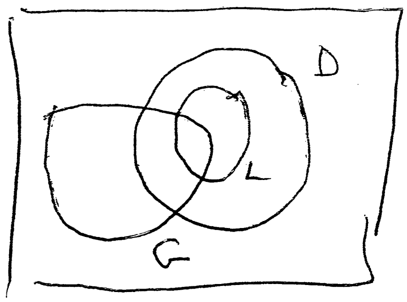
Math 461 HW6 Recap

On #1, the way to do it, really, is to draw a Venn diagram. That clarifies all relationships.

Some people wrote

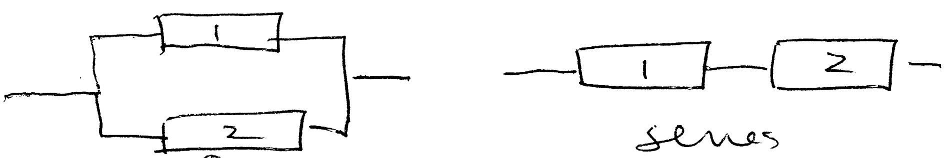


This indicates that $L \subseteq D$

It's ok, but not necessary.

I think I said everything I want to say on #2 on the solutions and in the notes. The key is to remember the ideas rather than memorize the formula.

Bonus Example, inspired by C+M HW3 Q3.



parallel   series

A = 1 works
B = 2 works

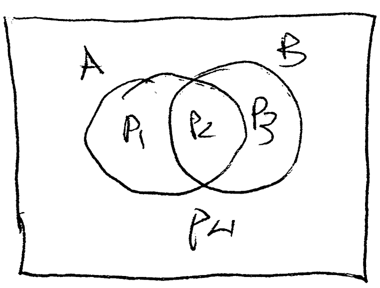
parallel works $\Leftrightarrow p(A \cup B)$ series works $\Leftrightarrow p(A \cap B)$

It is always the case that $p(A \cap B) \le p(A), p(B) \le p(A \cup B)$.

Suppose $p(A) = .8$ $\quad p(B) = .7$ and $P(2 \text{ fails} \mid 1 \text{ fails}) = p$.

Note that $A + B$ are indep $\Leftrightarrow A^c, B^c$ are independent $\Leftrightarrow$

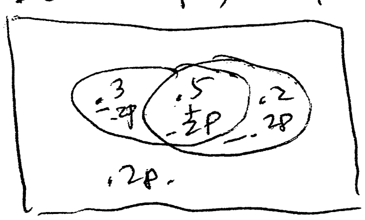
$p = p(2 \text{ fails} \mid 1 \text{ fails}) = P(2 \text{ fails}) = 1 - .7 = .3$



Given: $P_1 + P_2 + P_3 + P_4 = 1$
$P_1 + P_2 = .8 \Leftrightarrow P_3 + P_4 = .2$
$P_2 + P_3 = .7 \Leftrightarrow P_1 + P_4 = .3$

$$p = \frac{P(A^c \cap B^c)}{P(A^c)} = \frac{P_4}{P_3 + P_4} = \frac{P_4}{.2}$$

So $P_4 = p(.2)$, $P_3 = .2 - P_4 = (.2)(1-p)$, $\quad P_2 = .7 - P_3 = .7 - (.2)(1-p)$
$= .5 + .2p$, $\quad P_1 = .8 - P_2 = .3 - .2p$



So $P(A \cup B) = P_1 + P_2 + P_3 = 1 - .2p$
$P(A \cap B) = P_2 = .5 + .2p$