Math 461
HW 10
Recap

1. It is nearly always useful to know that the complement of "≥ 1" is "0" when counting things. An alternative solution to e) is

$$\sum_{k=1}^{6} \text{Prob}(0\ 7\text{'s and } k \text{ hearts}) = \sum_{k=1}^{6} \frac{\binom{12}{k}\binom{36}{6-k}}{\binom{52}{6}}$$

Throw out the 7's – there are 12 hearts + 36 non hearts, but don't divide by $\binom{48}{6}$. The universe is still 6-card hands out of 52.

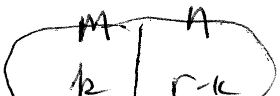
2. Lots of people started with $k=1$ for some reason.

3. Done in class. But ... if you get an expected value of $\frac{3}{8}$ for a random variable whose value is always ≥ 1 you should know something is wrong.

Bonus proof of identity.

$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$

(a) Combinatorial: You are choosing a set of r objects from a set of $m+n$. Split the big set into pieces of size m and n and count the number of ways of choosing k from the first set and, necessarily, $r-k$ from the second.



(b) Generating functions (as the term is usually used).

We have $(1+x)^N = \sum_{i=0}^{N} \binom{N}{i} x^i$, so

$$\boxed{\sum_{i=0}^{m+n} \binom{m+n}{i} x^i = (1+x)^{m+n} = (1+x)^m (1+x)^n = \left(\sum_{k=0}^{m} \binom{m}{k} x^k\right)\left(\sum_{l=0}^{n} \binom{n}{l} x^l\right)}$$

The coefficient of x^r on the left side occurs when $i=r$: and is $\binom{m+n}{r}$. The coefficients of x^r on the right side is $\binom{m}{k}\binom{n}{l}$ for all (k,l) where $k+l=r$. Adding them up, with $l=r-k$, gives us $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k}$