## 2.1 EXERCISES

**4.** A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume $V$ of water remaining in the tank (in gallons) after $t$ minutes.

| $t$ (min) | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| $V$ (gal) | 694 | 444 | 250 | 111 | 28 | 0 |

(a) If $P$ is the point (15, 250) on the graph of $V$, find the slopes of the secant lines $PQ$ when $Q$ is the point on the graph with $t = 5$, 10, 20, 25, and 30.

(b) Estimate the slope of the tangent line at $P$ by averaging the slopes of two secant lines.

(c) Use a graph of the function to estimate the slope of the tangent line at $P$. (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)

**5.** If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet $t$ seconds later is given by $y = 40t - 16t^2$.

(a) Find the average velocity for the time period beginning when $t = 2$ and lasting

(i) 0.5 second (ii) 0.1 second

(iii) 0.05 second (iv) 0.01 second

(b) Estimate the instantaneous velocity when $t = 2$.

6. If a rock is thrown upward on the planet Mars with [illegible]

2.2

1. Explain in your own words what is meant by the equation $\lim_{x \to 2} f(x) = 5$

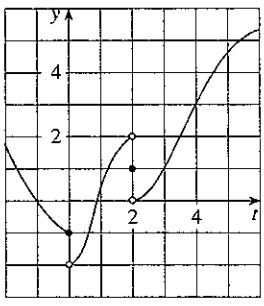
Is it possible for this to be true and yet $f(2) = 3$? Explain

**7.** For the function $g$ whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t \to 0^-} g(t)$ (b) $\lim_{t \to 0^+} g(t)$ (c) $\lim_{t \to 0} g(t)$

(d) $\lim_{t \to 2^-} g(t)$ (e) $\lim_{t \to 2^+} g(t)$ (f) $\lim_{t \to 2} g(t)$

(g) $g(2)$ (h) $\lim_{t \to 4} g(t)$




**13–16** Sketch the graph of an example of a function $f$ that satisfies all of the given conditions.

**15.** $\lim_{x \to 3^+} f(x) = 4$, $\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to -2} f(x) = 2$, $f(3) = 3$, $f(-2) = 1$

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**11–30** Evaluate the limit, if it exists.

**11.** $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

**12.** $\lim_{x \to -4} \dfrac{x^2 + 5x + 4}{x^2 + 3x - 4}$

**13.** $\lim_{x \to 2} \dfrac{x^2 - x + 6}{x - 2}$

**14.** $\lim_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

**15.** $\lim_{t \to -3} \dfrac{t^2 - 9}{2t^2 + 7t + 3}$

**16.** $\lim_{x \to -1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

**17.** $\lim_{h \to 0} \dfrac{(4 + h)^2 - 16}{h}$

**18.** $\lim_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$

**19.** $\lim_{x \to -2} \dfrac{x + 2}{x^3 + 8}$

**20.** $\lim_{h \to 0} \dfrac{(2 + h)^3 - 8}{h}$

**21.** $\lim_{t \to 9} \dfrac{9 - t}{3 - \sqrt{t}}$

**22.** $\lim_{h \to 0} \dfrac{\sqrt{1 + h} - 1}{h}$

**23.** $\lim_{x \to 7} \dfrac{\sqrt{x + 2} - 3}{x - 7}$

**24.** $\lim_{x \to -1} \dfrac{x^2 + 2x + 1}{x^4 - 1}$

**25.** $\lim_{t \to -4} \dfrac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

**26.** $\lim_{t \to 0} \left( \dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

**27.** $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

**28.** $\lim_{h \to 0} \dfrac{(3 + h)^{-1} - 3^{-1}}{h}$

**29.** $\lim_{t \to 0} \left( \dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t} \right)$

**30.** $\lim_{x \to -4} \dfrac{\sqrt{x^2 + 9} - 5}{x + 4}$

**35.** If $4x - 9 \le f(x) \le x^2 - 4x + 7$ for $x \ge 0$, find $\lim_{x \to 4} f(x)$.

36. If $2x \le g(x) \le x^4 - x^2 + 2$ for all $x$, evaluate $\lim_{x \to 1} g(x)$.

Remember - you can always do more problems than those assigned