Math 313 **Homework 3** **Due Fri., Feb. 20, 2004**

Since the counting approach of Chapter 3 is central to the rest of the course, I've decided to make this homework mainly an elaboration of the last one. As a bonus, all the graded problems come from old tests. There are some playing-card questions, a topic which is not prominent in the book. You should think of a **standard deck of cards** to be the cartesian product $S \times K$, where S is the set of *suits* – that is, {hearts, clubs, diamonds, spades} – and K is the set of *kinds* – that is, {A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K}. A **hand of k cards** is an (unordered) combination of k cards chosen from the deck of 52. The **probability** of a hand of a particular type is the ratio of the number of hands of that type to the number of hands with that many cards. No special knowledge of card playing will be assumed.

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1. – (ungraded) §3.6 – 23.

2. – (ungraded) §5.8 – 6.

3. – (ungraded) §3.6 – 9.

4. ($\mathcal{E}$) Neo-retro-ambient-folk-metal guitarist Ace Spitalny knows eight different chords. How many different four-chord patterns (order counts!) can Ace play if (a) No restriction is placed on the chords? (b) The chords must be different? (c) No chord can be played twice in a row?

5. ($\mathcal{E}$) How many ways can the letters T, E, N, N, E, S, S, E, E be arranged so that no two E's are consecutive? (It's a good idea to place the E's first.)

6. ($\mathcal{E}$) A **phone number** is a 7-digit decimal number whose first digit is not "0". A number is **palindromic** if it reads the same forwards and backwards. How many palindromic phone numbers are there? (E.g. "3492943" is a palindromic phone number.)

7. ($\mathcal{E}$) How many poker hands (5 cards) can be chosen from a standard deck, so that there is at least one card from each suit?

8. ($\mathcal{E}$) How many poker hands contain cards of 5 different values and so that four of the cards come from a red suit (hearts, diamonds) and one comes from a black suit (clubs, spades)?

9. ($\mathcal{E}$) Evaluate the following binomial expression. You may use any relevant binomial identity:

$$\sum_{k=0}^{353} k \binom{13}{k} \binom{15}{11-k}.$$

10. ($\mathcal{E}$) Back in the 1950's, the Beat Poets used to play the solitaire game of Beckett. You deal yourself 7 cards from a standard deck. If you obtain 3 cards from one suit, 2 cards from another suit and 1 card each from the remaining two suits, you win. Otherwise, you howl your despair at the shallow emptiness of modern existence. What is the probability that you win?