Math 313 **Homework 4** **Due Fri., Mar. 5, 2004**

Despite the impression you may have gotten from class, it **is** acceptable to prove a problem in only one way!

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1. – (ungraded) §5.8 – 17.

2. – (ungraded) §5.8 – 20.

3. – (ungraded) §5.8 – 27.

4. – §5.8 – 18.

5. – §5.8 – 30.

6. – §5.8 – 37.

7. – §6.6 – 2.

8. Evaluate the sum

$$\sum_{k=-\infty}^{\infty} \binom{n}{k} \cdot \binom{n}{k-1}$$

as a "closed" function of n. (Hint: follow the methodology we used to compute $\sum \binom{n}{k}^2$, taking into account one small, but significant, difference.)

9. ($\mathcal{E}$) Let

$$a_n = \sum_{k=0}^{\infty} \binom{n-k}{k}.$$

For example,

$$a_5 = \binom{5}{0} + \binom{4}{1} + \binom{3}{2} + \binom{2}{3} + \binom{1}{4} + \cdots = 1 + 4 + 3 + 0 + 0 + \cdots = 8.$$

Show that $a_0 = a_1 = 1$ and, for $n \geq 2$, prove that $a_n = a_{n-1} + a_{n-2}$. (This is an alternate expression for the Fibonacci numbers, which we'll be discussing later.)

10. ($\mathcal{E}$) Determine the number of ways to arrange the letters C,O,M,B,I,N,A,T,O,R,I,C,S which do **not** contain consecutive letters spelling any of the patterns "ROMANTIC", "CORN" or "TACO".