

1. – (ungraded) §8.5 – 6.

2. – (ungraded) §8.5 – 15.

3. – (ungraded) §8.5 – 26a.

4. – §8.5 – 7.

5. – §8.5 – 8.

6. – §8.5 – 19.

7. – §8.5 – 26b,c,d.

8. (\mathcal{E}) Let h_n denote the number of binary sequences of length n so that no three consecutive terms are the same; that is, “000” and “111” do not occur. Express h_n in terms of the Fibonacci numbers – $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. (Hint 1: clearly, $h_1 = 2$ and $h_2 = 4$, since no “bad” patterns of length 3 can exist!) (Hint 2: For $n \geq 2$, let a_n , b_n , c_n and d_n denote the valid strings of length n ending in “00”, “01”, “10” and “11” respectively. Consider what can follow them.)

9. (\mathcal{E}) The sequence (a_n) satisfies the recurrence

$$a_n = 4 \sum_{k=0}^n a_k a_{n-k}, \quad n \geq 2$$

with initial conditions $a_0 = 0$, $a_1 = 1$. Its generating function is given by

$$F(x) = \sum_{n=0}^{\infty} a_n x^n.$$

Determine carefully a quadratic equation satisfied by F and solve for F , giving a specific choice of sign in the “ \pm ”. **You do not have to solve for a_n !**

10. (\mathcal{E}) A $2 \times n$ strip of squares is to be covered with red, white and blue 1×2 dominos. For patriotic reasons, red and white dominos can only be horizontal, whereas blue dominos can be horizontal or vertical. Let a_n denote the number of ways of doing this. Determine a_1 , a_2 and a recurrence for a_n . You do not have to solve for a_n , but you should find a number α with the property that $\lim_{n \rightarrow \infty} \frac{a_n}{\alpha^n}$ exists and lies in $(0, \infty)$.