

1. – (ungraded) §8.5 – 6.

2. – (ungraded) §8.5 – 15.

3. – (ungraded) §8.5 – 26a.

4. – §8.5 – 7.

5. – §8.5 – 8.

6. – §8.5 – 19.

7. – §8.5 – 26b,c,d.

8. ( $\mathcal{E}$ ) Let  $h_n$  denote the number of binary sequences of length  $n$  so that no three consecutive terms are the same; that is, “000” and “111” do not occur. Express  $h_n$  in terms of the Fibonacci numbers –  $F_0 = 0$ ,  $F_1 = 1$ ,  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 2$ . (Hint 1: clearly,  $h_1 = 2$  and  $h_2 = 4$ , since no “bad” patterns of length 3 can exist!) (Hint 2: For  $n \geq 2$ , let  $a_n$ ,  $b_n$ ,  $c_n$  and  $d_n$  denote the valid strings of length  $n$  ending in “00”, “01”, “10” and “11” respectively. Consider what can follow them.)

9. ( $\mathcal{E}$ ) The sequence  $(a_n)$  satisfies the recurrence

$$a_n = 4 \sum_{k=0}^n a_k a_{n-k}, \quad n \geq 2$$

with initial conditions  $a_0 = 0$ ,  $a_1 = 1$ . Its generating function is given by

$$F(x) = \sum_{n=0}^{\infty} a_n x^n.$$

Determine carefully a quadratic equation satisfied by  $F$  and solve for  $F$ , giving a specific choice of sign in the “ $\pm$ ”. **You do not have to solve for  $a_n$ !**

10. ( $\mathcal{E}$ ) A  $2 \times n$  strip of squares is to be covered with red, white and blue  $1 \times 2$  dominos. For patriotic reasons, red and white dominos can only be horizontal, whereas blue dominos can be horizontal or vertical. Let  $a_n$  denote the number of ways of doing this. Determine  $a_1$ ,  $a_2$  and a recurrence for  $a_n$ . You do not have to solve for  $a_n$ , but you should find a number  $\alpha$  with the property that  $\lim_{n \rightarrow \infty} \frac{a_n}{\alpha^n}$  exists and lies in  $(0, \infty)$ .