Math 347 **Homework 1** **Due Wed., Sept. 10, 2003**

Reminder: the ungraded homework is still assigned – the answers are in the back, and these may/will show up on exams. The last two problems are harder, and are intended for graduate students trying to get 1.0U credit. You are all invited to try them if you like. The symbol ($\mathcal{E}$) before a problem means that it, or something very much like it, has appeared on an exam. I'm still working with the course TA to make an office hour schedule, but you should ask questions in class about the homework or send me email.

1. – (ungraded) 1.7.

2. – (ungraded) 2.3.

3. – (ungraded) 4.3 – a,c,e,g,i,k,m,p,q,s,u,w.

4. – ($\mathcal{E}$) Prove the following formula by mathematical induction: for $x \neq \pm 1$ and $n \geq 1$,

$$\sum_{k=1}^{n} \left(x^k - x^{2k}\right) = \frac{x(1-x^n)(1-x^{n+1})}{1-x^2}.$$

5. – 4.3 – b,d,f,h,j,l,n,p,r,t,v. (Note: *for this problem only*, it is ok to write down an answer without an explanation.)

6. – ($\mathcal{E}$) Suppose a and b are irrational real numbers and $a < b$. Prove that there exists an irrational number c so that $a < c < b$. To receive full credit, you should explain why c is irrational.

7. – 4.14a.

8. – 4.16.

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9. – Show that each of the following real numbers is irrational by the Rational Zeros Theorem. (Yes, you have to construct the polynomial first.)
   a. $\sqrt{2} - \sqrt{3}$;
   b. $\sqrt{2} + \sqrt{3} + \sqrt{5}$;
   c. $\sqrt{2} + \sqrt[3]{2}$.

10. – Compute (with proof)
   a. $\inf\{n + \frac{9}{n} \ : \ n \in \mathbf{N}\}$;
   b. $\sup\{n + \frac{9}{n} \ : \ n \in \mathbf{N}\}$;
   c. $\inf\{x^2 + (xy-1)^2 \ : \ x, y \in \mathbf{R}\}$. (Hint: calculus is not especially helpful here!)