Math 347 **Homework 10** **Due Wed., Nov. 19, 2003**

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1. – 28.1 b,c,d,e (ungraded).

2. – 28.3 b (ungraded).

3. – 29.1 a,c (ungraded).

4. – 28.2 a.

5. – 28.6.

6. – 29.10.

7. – ($\mathcal{E}$) Suppose f is a continuous function on $[0,2]$ and f is differentiable on $(0,2)$. Suppose further that

$$4 = \sup\{f(x) : x \in [0,2]\}, \qquad 1 = \inf\{f(x) : x \in [0,2]\}.$$

Let $M = \sup\{f'(x) : x \in (0,2)\}$. What does this say about M? (Hint: it's an inequality of the form $M \geq r$ or $M \leq r$ for some specific real number r.)

8. – ($\mathcal{E}$) Let $F(x)$ be a continuous, strictly increasing function defined on $[0,1]$ so that

$$\frac{x}{2} \leq F(x) \leq 2x$$

for all $x \in [0,1]$. For $x \in [0,1]$, let $f_n(x) = F(x^n)$. (As familiar examples, if $F(x) = x$, then $f_n(x) = x^n$; if $F(x) = x^n$, then $f_n(x) = \frac{x^n}{1+x^n}$.)

a. Compute $\lim f_n(x)$ for $x \in [0,1)$ and for $x = 1$.

b. Show by the comparison test that $\sum f_n(x)$ converges for $x \in [0,1)$.

c. Show that the convergence in (b) is uniform on the interval $[0,1/2]$.

d. What about uniform convergence on $[0,1)$? Be specific in your answer!

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9. – 28.14.

10. – 29.18.