Math 347 **(Last) Homework 11** **Due Wed., Dec. 3, 2003**

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1. – 32.3 (see #4) (ungraded).

2. – 33.5 (ungraded).

3. – 34.3 (ungraded).

4. – [Freebie] Have a good Thanksgiving and enjoy the point.

5. – 32.2.

6. – 34.2.

7. – 34.6.

8. – ($\mathcal{E}$) Let $f(x) = x^3 \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$. Using the ϵ/δ definition, prove that f is differentiable at 0. Assuming the familiar properties of trigonometric functions (for which you are not required to use the ϵ/δ definition), compute $f'(a)$ when $a \neq 0$. Is f' continuous at $x = 0$? Give a well-explained reason.

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9. – Suppose $|f(x)| \leq 17$ for $x \in [0,1]$ and f is continuous on $(0,1]$. We do not assume that f is continuous at $x = 0$! Show that f is integrable on $[0,1]$ by demonstrating that for every $\epsilon > 0$, there is a partition P of $[0,1]$ so that $U(f,P) - L(f,P) < \epsilon$. You might need the theorem that for every $a > 0$, f is integrable on $[a,1]$ since it is continuous there. (Note that a bounded function is not *necessarily* integrable on $[0,1]$.

10. – 34.8.

11. – Extra Credit. As we have seen, for $x \in [0,1)$,

$$x^n \geq \frac{x^n}{1+x^n} \geq \frac{x^n}{2} \implies \frac{1}{1-x} \geq \sum_{n=0}^{\infty} \frac{x^n}{1+x^n} \geq \frac{1}{2} \cdot \frac{1}{1-x}.$$

Find α so that $\lim_{x \to 1-} (1-x) \sum_{n=0}^{\infty} \frac{x^n}{1+x^n} = \alpha$. (If α exists, then $\alpha \in [\frac{1}{2}, 1]$ from the inequality.)

I know two different ways to do it that lie within the scope of this course; one involves a simple inequality related to the geometric series, the other involves a Riemann sum.