Math 347 **Homework 2** **Due Wed., Sept. 17, 2003**

Reminders: the ungraded homework is still assigned – the answers are in the back, and these may/will show up on exams. The last two problems are harder, and are intended for graduate students trying to get 1.0U credit. You are all invited to try them if you like. The symbol ($\mathcal{E}$) before a problem means that it, or something very much like it, has appeared on an exam. The TA for the course is Mr. Hua Tao. His office hours will be Tuesday 5:00 - 6:00 and Thursday 4:00 - 5:00 in 155 Altgeld Hall.

You may quote any theorem or example from class or the book ... PROVIDED that it has been proved there. It is not acceptable to quote an unproved homework problem as a step in proving an assigned homework problem!

1. – (ungraded) 7.3 a,c,e,g,i,k,m,o,q,s.

2. – (ungraded) 8.5. (This is an important result!)

3. – (ungraded) 9.13.

4. – 7.3 b,d,f,h,j,l,n,p,r,t.

5. – 8.2 b,e. (To receive full credit, your N should explicitly depend on ϵ.

6. – 9.6.

7. – ($\mathcal{E}$) Give a counterexample to the following assertion: If (a_n) is a sequence of real numbers and $\lim |a_{n+1} - a_n| = 0$, then (a_n) is convergent.

8. – ($\mathcal{E}$) let (a_n) be a sequence of real numbers satisfying the recurrence $a_{n+1} = \sqrt{6a_n - 8}$ for $n \ge 1$.

a. Suppose $\lim a_n$ exists and equals L. Determine the possible values for L.

b. Suppose $a_0 = 3$. Prove by induction (or any correct method) that $2 \le a_n \le 4$ for all $n \ge 0$.

c. Prove by induction (or any correct method) that $a_{n+1} > a_n$ for all $n \ge 0$.

d. Combine the previous parts to explain why $\lim a_n$ exists, and compute its value.

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9. – Suppose $0 < \alpha < 1$ and consider the sequence (a_n) defined by $a_0 = c$ and $a_{n+1} = \alpha a_n + \beta$ for $n \ge 0$, where β and c are fixed real constants. Prove by any correct method that (a_n) converges, and determine its limit.

10. – Two parts, unrelated.

a. Compute $\inf\{n + \frac{8}{n} \ : \ n \in \mathbf{N}\}$.

b. Determine with proof, by any correct method,

$$\lim \left(2(n^2+n)^{1/2} - (8n^3+n^2)^{1/3}\right).$$

Hint: you will probably want to consider an auxiliary sequence.